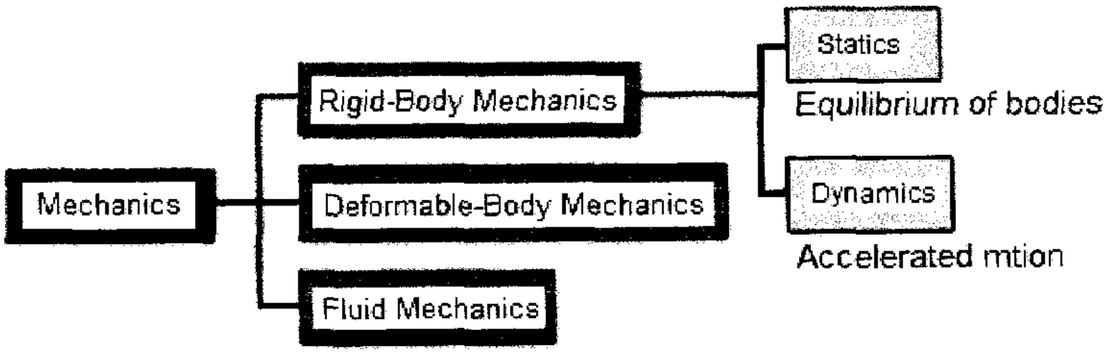
Introduction

Mechanics is the branch of physics concerned with the behavior of physical bodies at rest or in motion when subjected to forces or displacements, and the subsequent effects of the bodies on their environment. It can be divided to the following:



Statics is the study of bodies that are at rest or move with constant velocity. We can consider statics as a special case of dynamics, in which the acceleration is zero

Fundamental Concepts:

Length: Length is the quantity used to describe the position of a point in space relative to another point.

Time: Time is the interval between two events.

Mass: The amount of matter contained in a body. The mass of a body determines both the action of gravity on the body, and the resistance to changes in motion. This resistance to changes in motion is referred to as inertia.

Force: Force is the action of one body on another. Force may or may not be the result of direct contact between bodies such as the gravitational, and electromagnetic

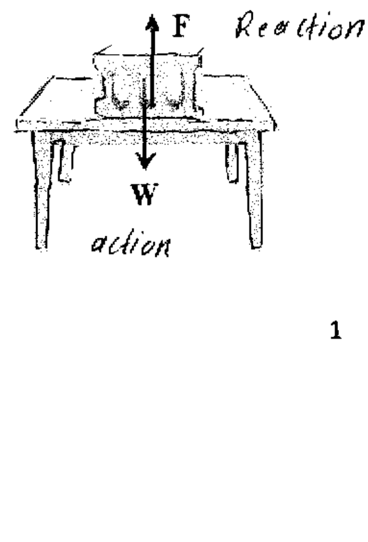
Rigid Body: Rigid body - a body is considered rigid when the relative movements between its parts are negligible.

Newton's 3 Fundamental Laws

1st Law - A particle remains at rest or continues to move in a straight line with a constant speed if there is no unbalanced force acting on it (resultant force = 0).

2nd Law - The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.

$F = m \cdot a$ where: F: force
m: Mass
a: Acceleration



3rd Law - The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and act along the same line of action (Collinear).

Units:

		SI		U.S.	
Mass	M	kilogram	kg	slug	
Length	L	meter	m	feet	ft
Force	F	Newton	N	pound	lb
Time	T	second	s	second	sec

Mass vs. Weight:

The weight of a body is the force exerted on the body due to gravitational attraction of the Earth.

W= m g

W is weight

m is mass

g is acceleration due to gravity $= 9.81\ m/s^2$ (SI units)

Also $g = 32.2\ ft/s^2$ (English units)

$1\ N = (1\ kg)\ (1\ m/s^2)$

$1\ lb = (1\ slug)\ (1\ ft/sec^2)$

Weight is a force.

<u>Example:</u> The weight of 1 kg Mass is:

$W = mg$

$W = (1\ kg)(9.81\ m/\ s^2)$

$W = 9.81\ N$

Unit conversion:

I lb	=	4.4482 N
كتلة 1slug	=	$1\ lb\ s^2/ft = 14.5938\ kg$
1 ft	=	0.3048 m
1 ft	=	12 in
1mile	=	5,280 ft
1 kip	=	1,000 lb
1 ton	=	2,000 lb

Scalars and Vectors

There are two types of quantities in physics Scalars and Vectors.

Scalars quantities has only *magnitude* such as length, area, volume, mass, density, temperature, energy, ..etc

Vectors has *magnitude* + *direction* such as: displacement, direction velocity, force,..etc

Vectors:

A vector quantity, $\mathbf{V}$ is drawn as in Figure 1.1, a line segment with an arrow head to indicate direction. The length of the line segment indicates the vector magnitude, $|V|$ (printed as V), using a convenient scale.

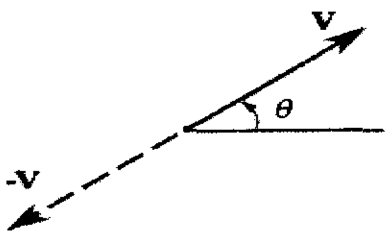
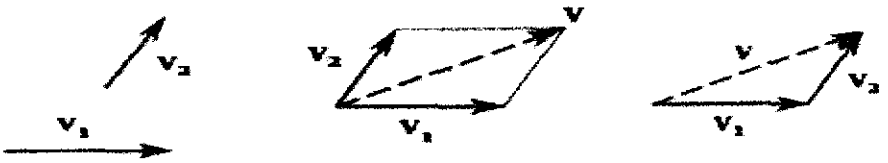


Figure 1.1: Vector drawing and labeling conventions

A vector's direction may be described by an angle, given from a known origin and line of reference, as in Figure 1.1.

Using Vectors



(a) (b) (c)

Figure 1.2: Parallelogram law of combination

Vectors obey the parallelogram law of combination

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2, \qquad \mathbf{V}_1 + \mathbf{V}_2 = \mathbf{V}_2 + \mathbf{V}_1.$$

The *vector sum* is indicated by the addition sign between bold-faced vectors. This should not be confused with the *scalar sum* of the two vectors, $V_1 + V_2$. Because of the geometry of the parallelogram, clearly $V \neq V_1 + V_2$.

Typically we use *rectangular components* using x and y as coordinates.

(a) (b)

Figure 1.4: Parallelogram law of combination

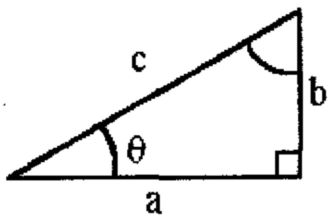
$$\theta = \tan^{-1}\frac{V_y}{V_x}$$

Triangle law can be used in calculating vectors angles which is as folowing:

For a right triangle:

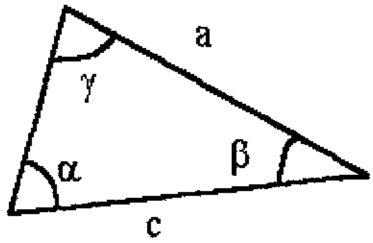
$a^2 + b^2 = c^2$

$\tan(\theta) = b/a, \qquad \sin(\theta) = b/c, \qquad \cos(\theta) = a/c$



For a general triangle:

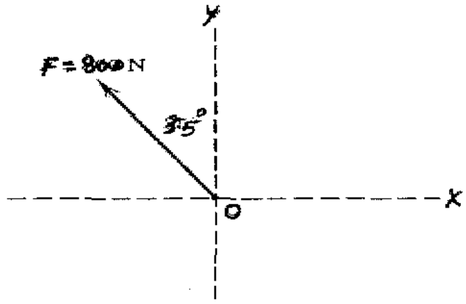
$\alpha+\beta+\gamma=180^o$



Sine law: $\dfrac{a}{\sin(\alpha)}=\dfrac{b}{\sin(\beta)}=\dfrac{c}{\sin(\gamma)}$

Cosine law: $c=\sqrt{a^2+b^2-2ab\cos(\gamma)}$

Example: The force **F** has a magnitude of 800 N. Express **F** as a vector in terms of the unit vectors **i** and **j**. Identify the *x* and *y* scalar components of F.



Solution:

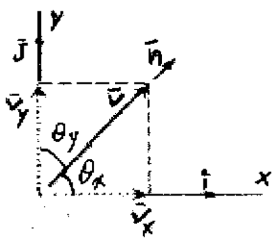
$$\begin{cases} F_x = -800\sin 35^\circ = -459 \text{ N} \\ F_y = 800\cos 35^\circ = 655 \text{ N} \end{cases}$$

$$\bar{F} = -459\,i + 655\,j \quad N$$

Unit Vector:

$$\bar{\mathbf{V}} = |\,\mathbf{V}\,|\,\bar{\mathbf{n}}$$

Magnitude ($|V|$) — Direction (dimensionless) ($\bar{n}$)



$\bar{n}$ = Unit vector in direction of $\bar{V}$

$$\bar{n} = \frac{\bar{V}}{|\bar{V}|} = \frac{V_x\hat{i}+V_y\hat{j}}{|\bar{V}|} = \frac{V_x}{|\bar{V}|}\hat{i} + \frac{V_y}{|\bar{V}|}\hat{j}$$

$$= \cos\theta_x\hat{i} + \cos\theta_y\hat{j}$$

$\dfrac{V_x}{|\bar{V}|} = \cos\theta_x$ = direction cosine $\quad |V| = \sqrt{V_x^2+V_y^2}$

$$\cos\theta_x^2 + \cos\theta_y^2 = 1$$

Taking V as the vector magnitude, a vector may be described as its magnitude multiplied by its unit vector. The value of **V** does not change as a result, because the *unit vector* indicates the proper direction and has a magnitude of one.

In order to describe a vector in 2 or 3 (orthogonal) dimensions, we use the unit vectors, **i**, **j**, and **k** to indicate a vector's direction towards x, y, and z, respectively (as seen in Figure 1.5).

$$\bar{\mathbf{V}} = \bar{V}x\,\mathbf{i} + \bar{V}y\,\mathbf{j} + \bar{V}z\,\mathbf{k}$$

$$|V| = \sqrt{V_x^2+V_y^2+V_z^2}$$

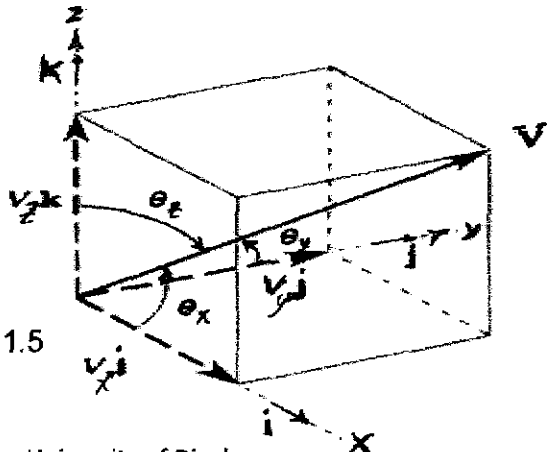


Figure 1.5

Example:

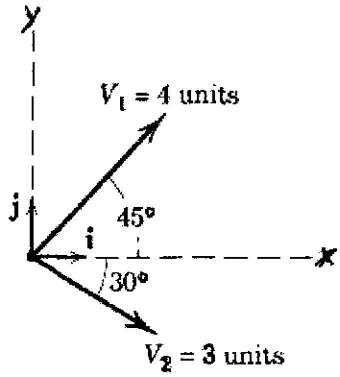
For the vectors $\overline{V}_1$ and $\overline{V}_2$ shown in the figure,

(a) determine the magnitude S of their vector sum $\overline{S} = \overline{V}_1 + \overline{V}_2$

(b) determine the angle α between $\overline{S}$ and the positive x-axis

(c) write $\overline{S}$ as a vector in terms of the unit vectors **i** and **j** and then write a unit vector $\overline{n}$ along the vector sum $\overline{S}$

(d) determine the vector difference $\overline{D} = \overline{V}_1 - \overline{V}_2$

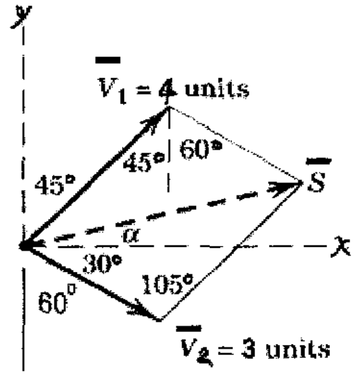


Solution:

(a) We construct to scale the parallelogram shown in Fig. a for adding V_1 and V_2. Using the law of cosines, we have

$$S^2 = 3^2 + 4^2 - 2(3)(4)\cos 105^\circ$$

$$S = 5.59 \text{ units} \qquad \textit{Ans.}$$



(a)

(b) Using the law of sines for the lower triangle, we have

$$\frac{\sin 105^\circ}{5.59} = \frac{\sin(\alpha + 30^\circ)}{4}$$

$$\sin(\alpha + 30^\circ) = 0.692$$

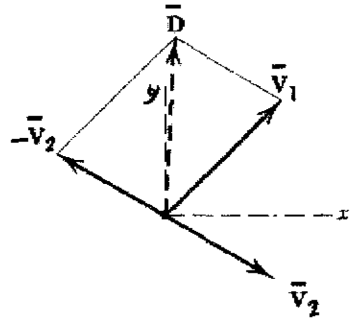
$$(\alpha + 30^\circ) = 43.8^\circ \qquad \alpha = 13.76^\circ \qquad \textit{Ans.}$$

(c) With knowledge of both S and α, we can write the vector **S** as

$$\overline{S} = S[\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha]$$

$$= S[\mathbf{i}\cos 13.76^\circ + \mathbf{j}\sin 13.76^\circ] = 5.43\mathbf{i} + 1.328\mathbf{j} \text{ units} \qquad \textit{Ans.}$$

Then $$\overline{n} = \frac{\overline{S}}{S} = \frac{5.43\mathbf{i} + 1.328\mathbf{j}}{5.59} = 0.971\mathbf{i} + 0.238\mathbf{j} \qquad \textit{Ans.}$$



(d) The vector difference $\overline{D}$ is

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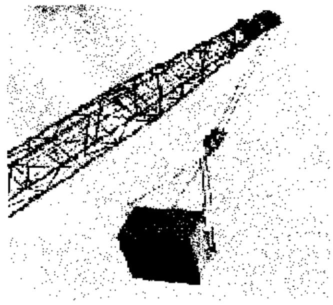
$$\overline{D} = \overline{V}_1 - \overline{V}_2 = 4(\mathbf{i}\cos 45^\circ + \mathbf{j}\sin 45^\circ) - 3(\mathbf{i}\cos 30^\circ - \mathbf{j}\sin 30^\circ)$$

$$= 0.230\mathbf{i} + 4.33\mathbf{j} \text{ units} \qquad \textit{Ans.}$$

The vector $\overline{D}$ is shown in Fig. b as $\overline{D} = \overline{V}_1 + (-\overline{V}_2)$.

Force



Force: is something which acts upon a body which is either a push or a pull.

Force is completely characterized by its **magnitude, direction,** and **point of application**, and therefore its vector.

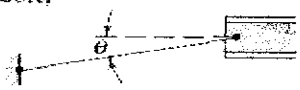
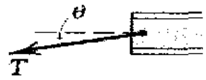
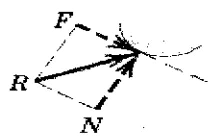
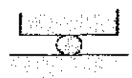
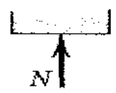
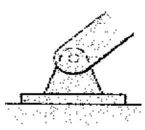
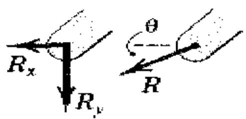
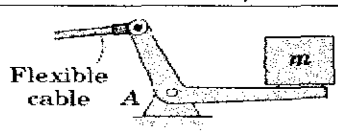
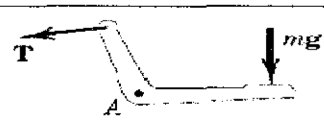
Free-body diagrams

A free body diagram is a sketch of the body and all the forces acting on it. They are termed fee-body diagrams because each diagram considers only the forces acting on the particular object considered.

Approach:

- Resolve force vectors in to appropriate components
- Isolate the body, remove all supports and connectors.
- Identify all EXTERNAL forces acting on the body.
- Make a sketch of the body, showing all forces acting on it.

Samples of Free Body Diagrams:

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible 	 Force exerted by a flexible cable is
2. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent F (frictional force) as well as a normal component N of the resultant
3. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
4. Pin connection 	Pin free to turn  Pin not free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
5. Bell crank supporting mass m with pin support at A. 	

Force analysis:

Resolution of forces into components

It is often to decompose a single force acting at some angle from the coordinate axes into perpendicular forces called *components*. The component of a force parallel to the x-axis is called the x-component, parallel to y-axis the y-component, and so on.

These forces, when acting together, have the same external effect on a body as the original force. They are known as **components**. Finding the components of a force can be viewed as the converse of finding a resultant.

Components of a Force in XY Plane:

$$F_x = F\cos\theta_x = F\sin\theta_y$$

$$F_y = F\sin\theta_x = F\cos\theta_y$$

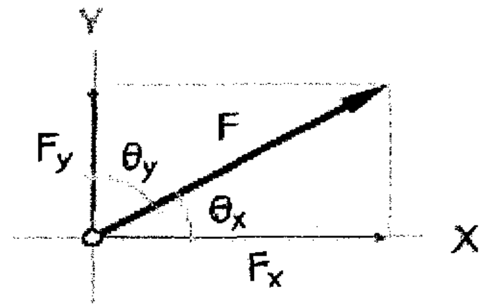
$$F = \sqrt{F_x^2 + F_y^2}$$

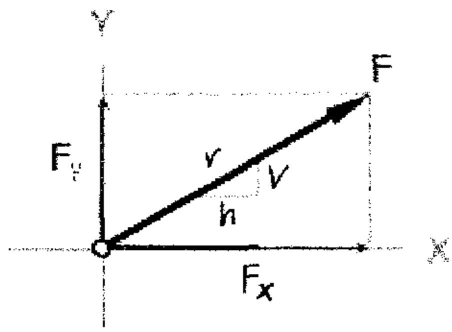
$$\tan\theta_x = \frac{F_y}{F_x}$$

$$r = \sqrt{h^2 + v^2}$$

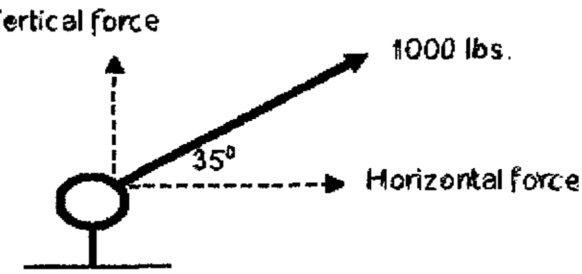
$$F_x = F(h/r)$$

$$F_y = F(v/r)$$





Most forces on inclined surfaces, or inclined forces are resolved by solving triangles. Another example of a force acting on an anchor is as follows:

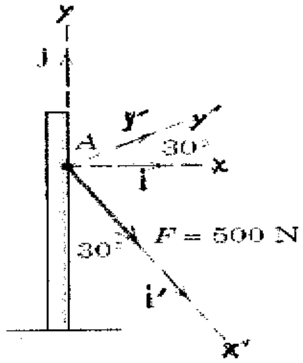


Vertical force = 1000 lbs(sin(35°))
= **574 lbs**

Horizontal force = 1000 lbs(cos(35°))
= **819 lbs**

Example:

The 500-N force **F** is applied to the vertical pole as shown. (1) Write **F** in terms of the unit vectors **i** and **j** and identify both its vector and scalar components. (2) Determine the scalar components of the force vector **F** along the x'- and y'-axes. (3) Determine the scalar components of **F** along the x- and y'-axes.



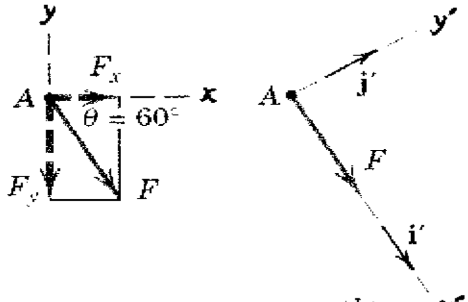
Solution:

Part (1). From Fig. *a* we may write **F** as

$$\overline{\mathbf{F}} = (F\cos\theta)\mathbf{i} - (F\sin\theta)\mathbf{j}$$

$$= (500\cos 60^\circ)\mathbf{i} - (500\sin 60^\circ)\mathbf{j}$$

$$= (250\mathbf{i} - 433\mathbf{j})\text{ N} \qquad \textit{Ans.}$$



(*a*) (*b*)

The scalar components are $F_x = 250$ N and $F_y = -433$ N. The vector components are $\mathbf{F_x} = 250\mathbf{i}$ N and $\mathbf{F_y} = -433\mathbf{j}$ N.

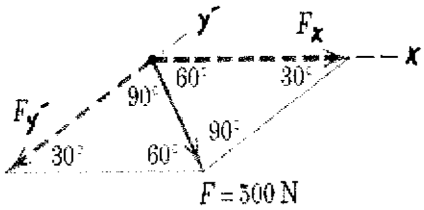
Part (2). From Fig. *b* we may write $\overline{\mathbf{F}}$ as $\overline{\mathbf{F}} = 500\mathbf{i}'$ N, so that the required scalar components are

$$F_{x'} = 500\text{ N} \qquad F_{y'} = 0 \qquad \textit{Ans.}$$

Part (3). The components of $\overline{\mathbf{F}}$ in the x- and y'-directions are nonrectangular and are obtained by completing the parallelogram as shown in Fig. *c*. The magnitudes of the components may be calculated by the law of sines. Thus,

$$\frac{|F_x|}{\sin 90^\circ} = \frac{500}{\sin 30^\circ} \qquad |F_x| = 1000\text{ N}$$

$$\frac{|F_{y'}|}{\sin 60^\circ} = \frac{500}{\sin 30^\circ} \qquad |F_{y'}| = 866\text{ N}$$



(*c*)

The required scalar components are then

$$F_x = 1000\text{ N} \qquad F_{y'} = -866\text{ N} \qquad \textit{Ans.}$$

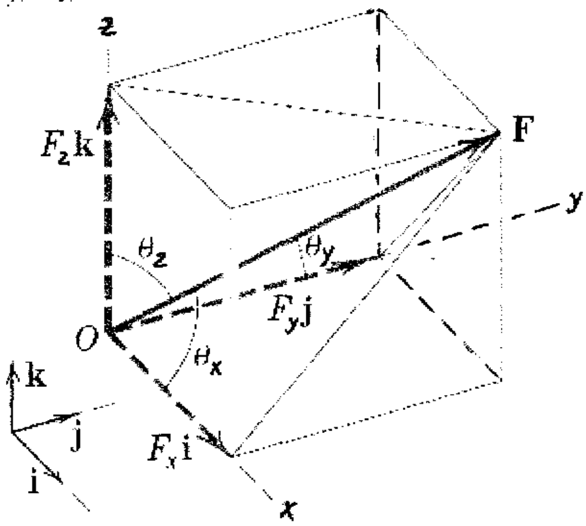
Components Force in 3D Space:

Many problems in mechanics require analysis in three dimensions, and for such problems it is often necessary to resolve a force into its three mutually perpendicular components. The force **F** acting at point O in Fig. 2/16 has the *rectangular components* F_x, F_y, F_z, where

$$F_x = F\cos\theta_x \qquad F = \sqrt{F_x^{\,2} + F_y^{\,2} + F_z^{\,2}}$$

$$F_y = F\cos\theta_y$$

$$F_z = F\cos\theta_z \qquad \bar{\mathbf{F}} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$



Specification by two angles which orient the line of action of the force.

Consider the geometry of the fig. We assume that the angles θ and ϕ are known. First resolve **F** into horizontal and vertical components.

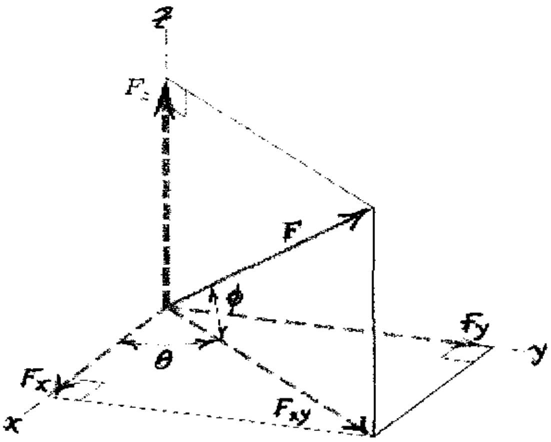
$$F_{xy} = F\cos\phi$$

$$F_z = F\sin\phi$$

Then resolve the horizontal component F_{xy} into x- and y-components.

$$F_x = F_{xy}\cos\theta = F\cos\phi\cos\theta$$

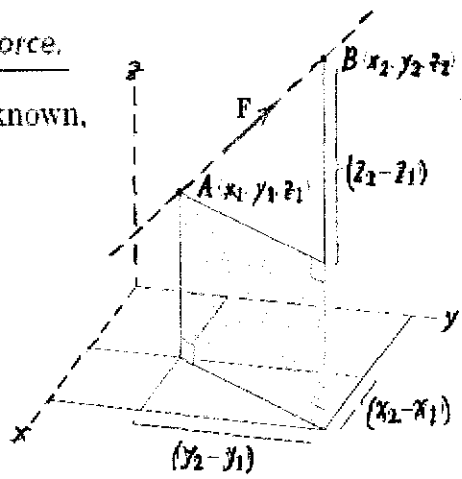
$$F_y = F_{xy}\sin\theta = F\cos\phi\sin\theta$$



Specification by two points on the line of action of the force.

If the coordinates of points A and B of Fig the figure are known, the force **F** may be written as

$$\mathbf{F} = F\mathbf{n}_F = F\frac{\overrightarrow{AB}}{\overline{AB}} = F\,\frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

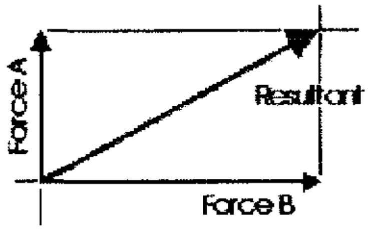


Where: $\bar{F}$: Vector

F: The magnitude of the vector

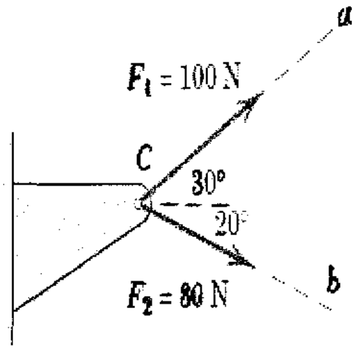
Resultant of force system:

The resultant is a representative force which has the same effect on the body as the group of forces it replaces. One can progressively resolve pairs or small groups of forces into resultants. Then another resultant of the resultants can be found and so on until all of the forces have been combined into one force. Resultants can be determined both graphically and algebraically.

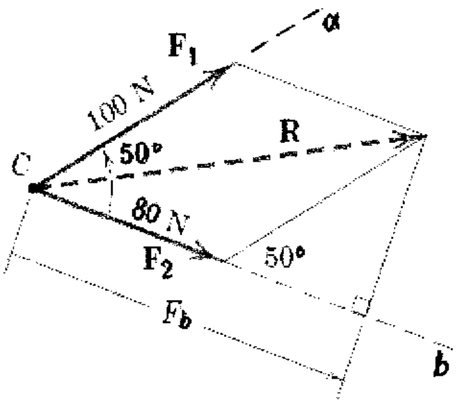


Example:

Forces $\mathbf{F_1}$ and $\mathbf{F_2}$ act on the bracket as shown. Determine the projection F_b of their resultant $\mathbf{R}$ onto the b-axis.



Solution:



The parallelogram addition of $\mathbf{F}_1$ and $\mathbf{F}_2$ is shown in the figure. Using the law of cosines gives us

$$R^2 = (80)^2 + (100)^2 - 2(80)(100)\cos 130^\circ \qquad R = 163.4 \text{ N}$$

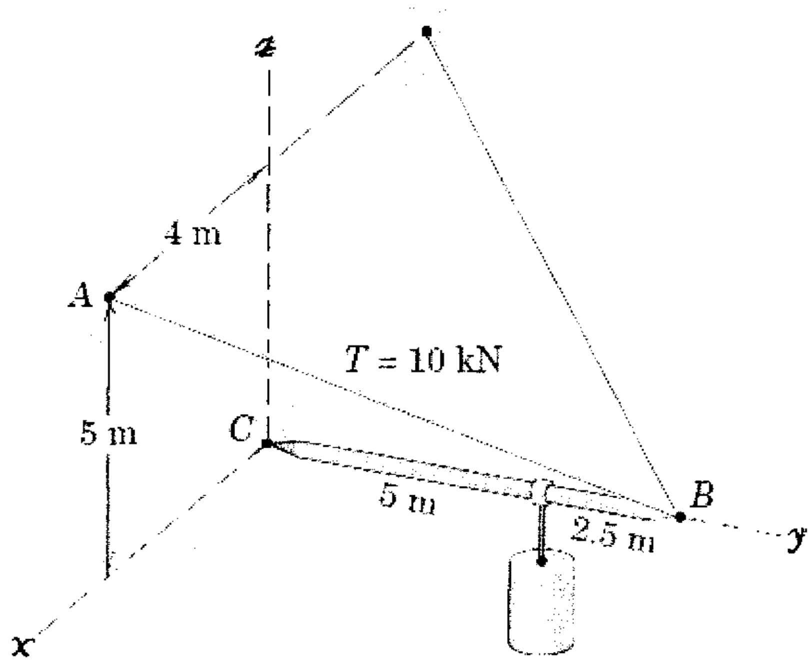
The figure also shows the orthogonal projection $\mathbf{F}_b$ of $\mathbf{R}$ onto the b-axis. Its length is

$$F_b = 80 + 100\cos 50^\circ = 144.3 \text{ N} \qquad \textit{Ans.}$$

Note that the components of a vector are in general not equal to the projections of the vector onto the same axes. If the a-axis had been perpendicular to the b-axis, then the projections and components of $\mathbf{R}$ would have been equal.

Example:

The tension in the supporting cable AB is 10 kN. Write the force which the cable exerts on the boom BC as a vector $\mathbf{T}$. Determine the angles θ_x, θ_y, and θ_z which the line of action of $\mathbf{T}$ forms with the positive x-, y-, and z-axes.



Solution:

$$\bar{T} = T\,\bar{n}_{AB} = 10\left[\frac{-4i + 7.5j + 5k}{(4^2 + (-7.5)^2 + 5^2)^{1/2}}\right]$$

$$= 10\,(-0.406\,i + 0.761\,j + 0.507\,k)\ \text{kN}$$ Ans.

$\cos\theta_x = -0.406$, $\theta_x = 66.1°$ Ans.

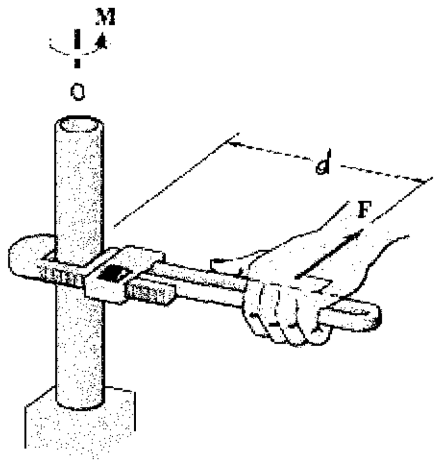
$\cos\theta_y = +0.761$, $\theta_y = 139.5°$ Ans.

$\cos\theta_z = -0.507$, $\theta_z = 59.5°$ Ans.

Moment, Coples and Equlibrium

Moment of a force:

The Moment of Force (F) about an axis through Point (o) or for short is the product of the magnitude of the force and the perpendicular distance between point (o) and the line of action of force (F)



$$M = F d$$

The units of a Moment are: $N{\cdot}m$ in the SI system

$ft{\cdot}lbs$ or $in{\cdot}lbs$ in the US Custor

M: Magnitude of the moment of **F** around point O

d: Perpendicular distance from O to the line of action of **F**

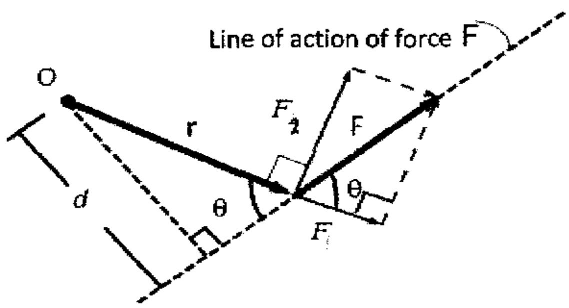
Direction of the moment in 2-D:

The direction of the moment is given by the right hand rule: Counter Clockwise (CCW) is out of the page, Clockwise (CW) is into the page.

CCW-out of the CW-into the page

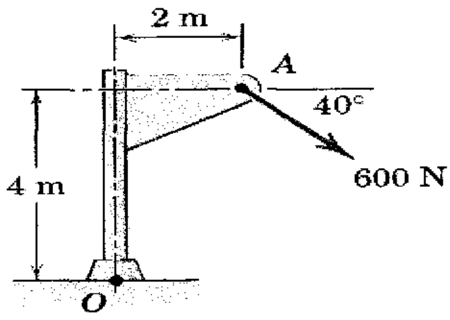
Moving a force along its line of action:

Moving a force along its line of action, results in a new force system which is equivalent to the original force system.



Note: moving a force along its line of action does not change its moment

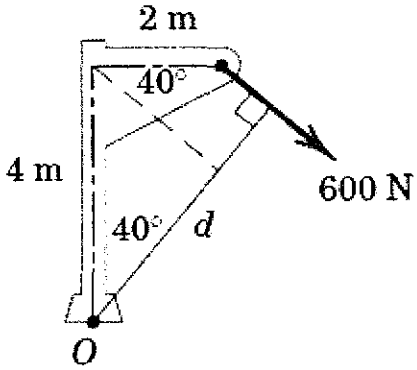
$$M = Fd = Fr \operatorname{Sin}(\theta)$$



Example:

Calculate the magnitude of the moment about the base point *O* of the 600-N force in five different ways.

Solution:



(I) The moment arm to the 600-N force is

$$d = 4\cos 40^\circ + 2\sin 40^\circ = 4.35 \text{ m}$$

By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

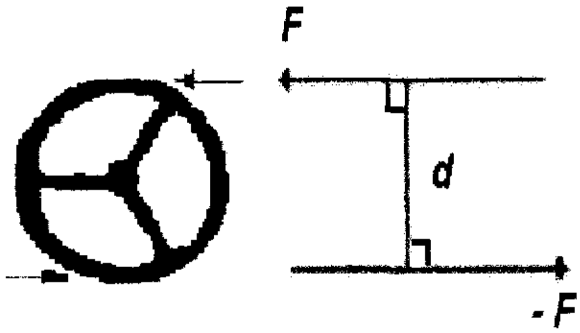
(II) Replace the force by its rectangular components at *A*

$$F_1 = 600\cos 40^\circ = 460 \text{ N}, \qquad F_2 = 600\sin 40^\circ = 386 \text{ N}$$

the moment becomes

$$M_O = 460(4) + 386(2) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

Moment of a Couple:

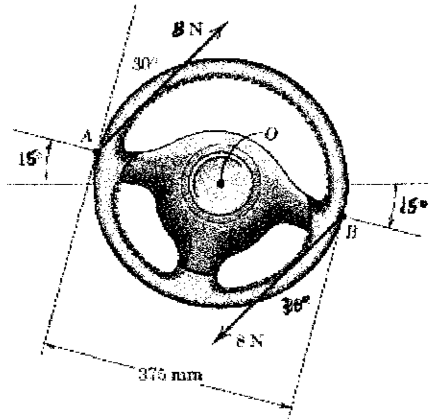


A couple can be defined as two parallel forces, having the same magnitude, opposite directions and separated by a perpendicular distance *d*

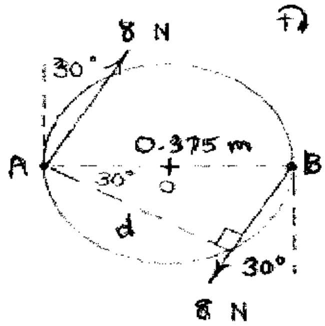
The moment of a couple is the product of the magnitude of one of the forces and the perpendicular distance between their lines of action. **M = F x d.** It has the units of kip-feet, pound-inches, KN-meter, etc.

Example:

During a steady right turn, a person exerts the forces shown on the steering wheel. Note that each force consists of a tangential component and a radially inward component. Determine the moment exerted about the steering column at O.



Solution:

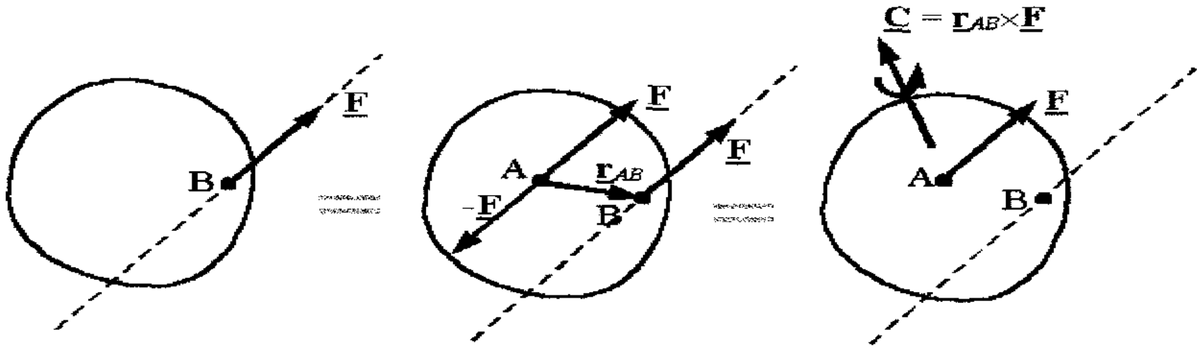


$M = Fd = 8\,(0.375\cos 30°)$

$= \underline{2.60\ N\cdot m\ \ CW}$

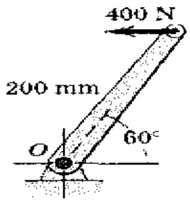
Moving a force off its line of action:

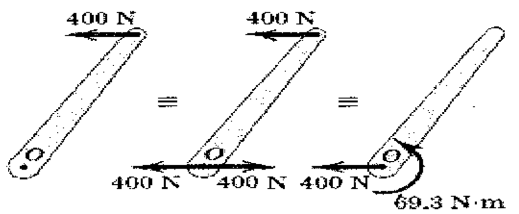
If a force is moved off its line of action, a couple must be added to the force system so that the new system generates the same moment as the old system.



Example:

Replace the horizontal 400-N force acting on the lever by an equivalent system consisting of a force at O and a couple.





Solution:

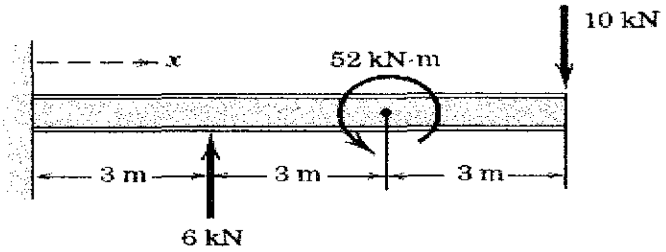
We apply two equal and opposite 400-N forces at O and identify the counterclockwise couple

$[M = Fd]$ $\qquad M = 400(0.200 \sin 60°) = 69.3\ \text{N}\cdot\text{m}$ $\qquad$ *Ans.*

Thus, the original force is equivalent to the 400-N force at O and the 69.3-N·m couple as shown in the third of the three equivalent figures.

Example:

Determine and locate the resultant **R** of the two forces and one couple acting on the I-beam.

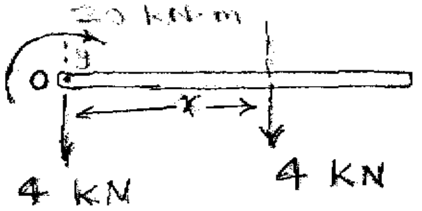


Solution:

Force – Couple system at point o:

$$R = \Sigma F = (6-10) = -4 \quad kN$$

$$\curvearrowleft + \; M_o = 6(3) - 10(9) + 52 = -20 \; kN \cdot m$$

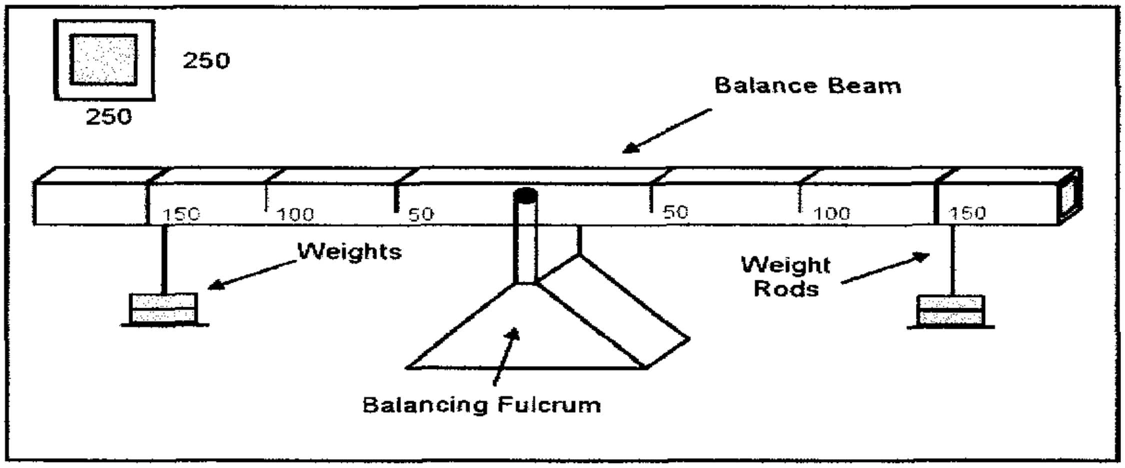


$$x = \frac{M_o}{R} = \frac{20}{4}$$

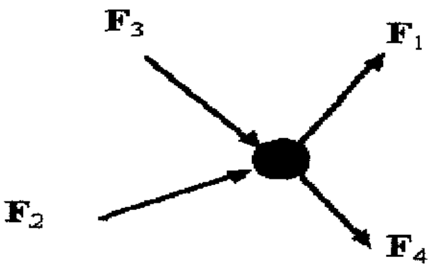
$$= 5 \; m \quad (\text{on beam})$$

Equilibrium:

A body is in equilibrium if the resultant of all the external forces and moments acting on the body is zero.



$$\sum \mathbf{F} = \mathbf{0}$$
$$\sum \mathbf{M} = \mathbf{0}$$



Equilibrium equations in component form:

In a rectangular coordinate system the equilibrium equations can be represented by three scalar equations:

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$

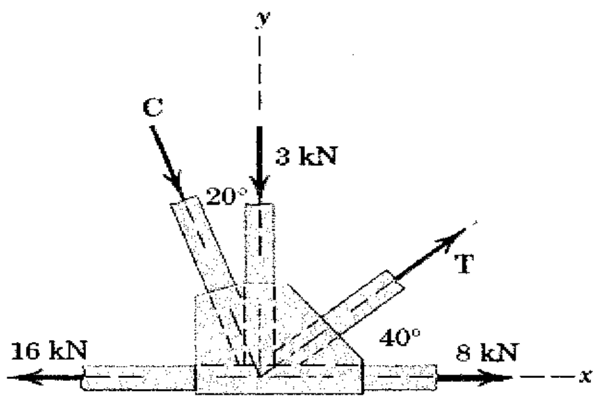
Sum of All Moments (Mz) = 0

(Sum of All Moments (My) = 0)

(Sum of All Moments (Mx) = 0)

Example:

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



The given sketch constitutes the free-body diagram of the isolated section of the joint in question and shows the five forces which are in equilibrium.

For the x-y axes as shown we have

$[\Sigma F_x = 0]$ $\qquad 8 + T\cos 40° + C\sin 20° - 16 = 0$

$$0.766T + 0.342C = 8 \qquad (a)$$

$[\Sigma F_y = 0]$ $\qquad T\sin 40° - C\cos 20° - 3 = 0$

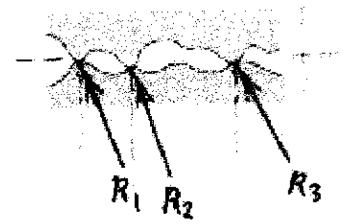
$$0.643T - 0.940C = 3 \qquad (b)$$

Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09\text{ kN} \qquad C = 3.03\text{ kN} \qquad \textit{Ans.}$$

Friction

Introduction



Friction is a force that resists the movement of two contacting surfaces that slide relative to one another. This force always acts tangent to the surface at the points of contact and is directed so as to oppose the possible or existing motion between the surfaces.

There are two types of friction: *dry friction* and *fluid friction*. Fluid friction applies to lubricated mechanisms.

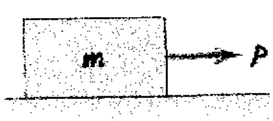
The present discussion is limited to dry friction between non-lubricated surfaces.

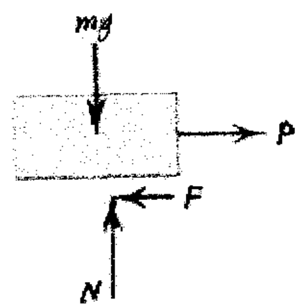
Dry friction:

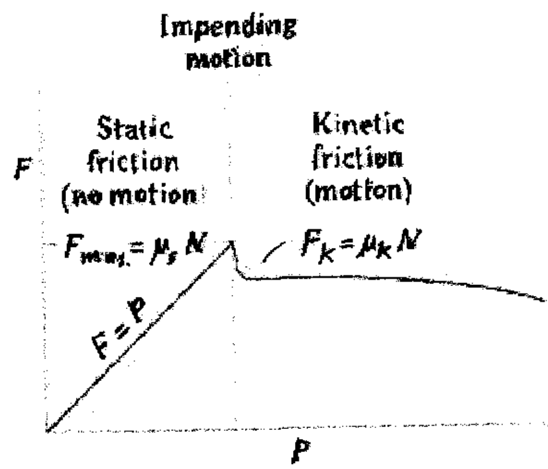
From experiment, its found that the frictional force:

- is proportional to the normal force
- is independent of the area of contact
- depends on whether the object is stationary or sliding

Three regions of Static – Motion Friction:







mg: weight of the block

N: Reaction of the weight

P: Small horizontal force applied to block.

F: is a static-friction force

While applying the horizontal force **P**, the block to remains stationary, in equilibrium, which is the stage of static friction:

$F \le F_{max.} \quad \rightarrow \quad F \le \mu_s N$ Where: μ_s coefficient of static friction

But when P increases, the static-friction force F increases until it reaches a maximum value Fmax.

$F = F_{max.} \quad \rightarrow \quad F = \mu_s N$

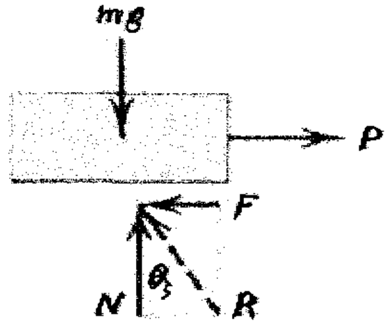
Further increase in **P** causes the block to begin to move. Once the body starts to slip, then its in the stage of has kinetic (dynamic or slipping) friction:

$F = \mu_k N$ Where μ_k : coefficient of kinetic friction

In general the coefficient of kinetic friction is smaller than the coefficient of static friction, which explains the initial difficulty of getting an object to slide.

$$\boxed{\mu_k < \mu_s}$$

The Resultant of the friction and normal forces;



$$R = \sqrt{N^2 + F^2}$$

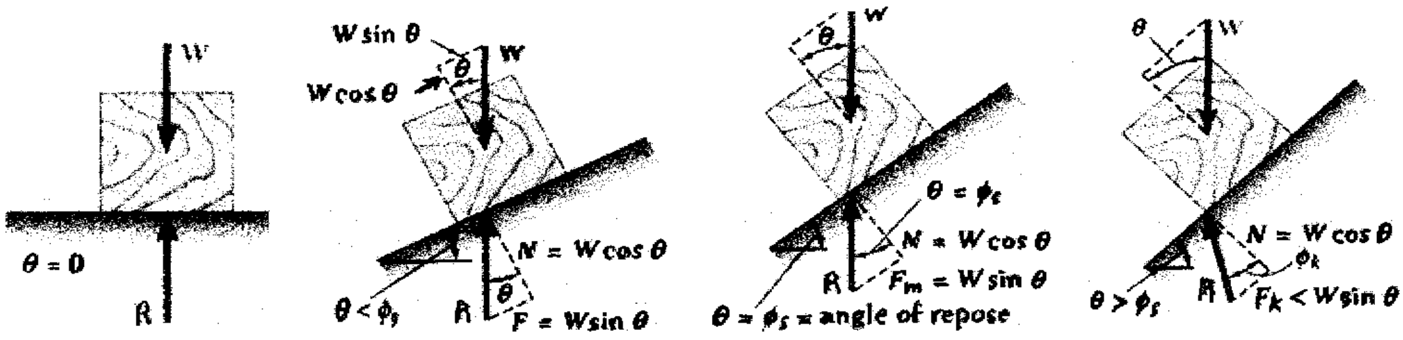
At the point of impending motion: $R = \sqrt{N^2 + F^2_{max}}$

$$\tan \phi_s = F/N = \mu_s N/N = \mu_s$$

ϕ_S is the angle of static friction

Gravity forces in inclined surfaces :

Consider block of weight *W* resting on board with variable inclination angle (θ)



- No friction
- No motion
- Motion impending
- Motion

When the friction force reaches the max value, the friction force reaches the max value then:

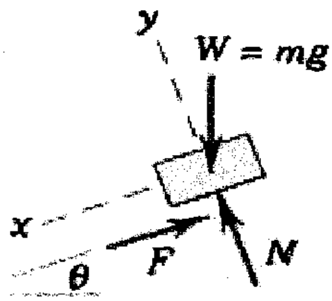
$$\tan \phi_s = \mu_s$$

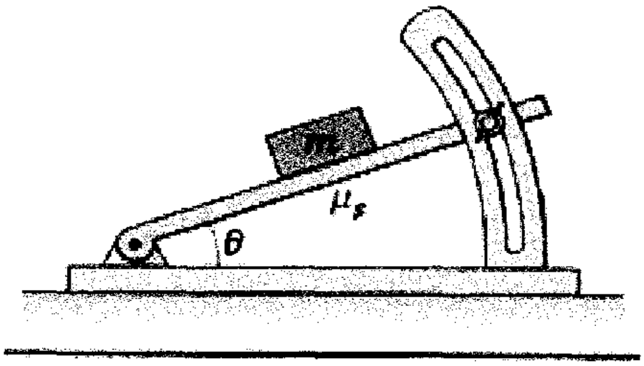
Some appropriate coefficient of friction:

	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminium on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25 - 0.5	0.2
Glass on glass	0.94	0.4
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

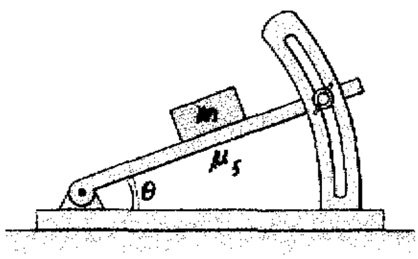
Example:

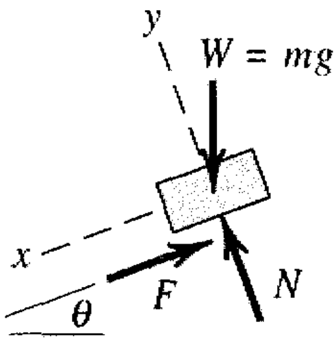
Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s.





Solution:



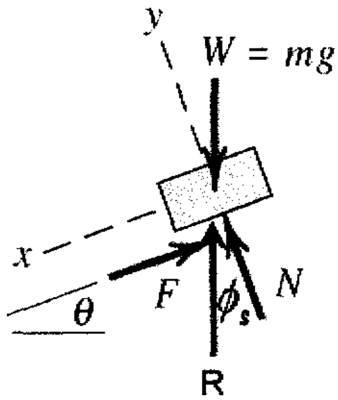


at the moment of slipping, friction is $F = \mu_s N$ upward

$[\sum F_y = 0]$ $\quad N - mg\cos\theta = 0$

$[\sum F_x = 0]$ $\quad \mu_s N - mg\sin\theta = 0$

$\mu_s = \tan\theta$ or $\theta = \tan^{-1}\mu_s$

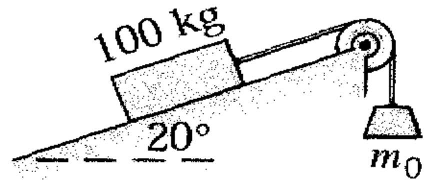


when the friction force reaches max value, $\tan\phi_s = \mu_s$

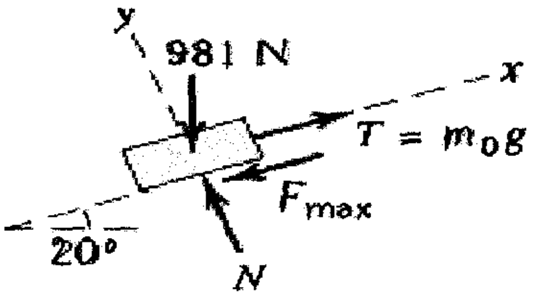
by equilibrium, R = W and $\phi_s = \theta \quad \therefore \theta = \tan^{-1}\mu_s$

Example:

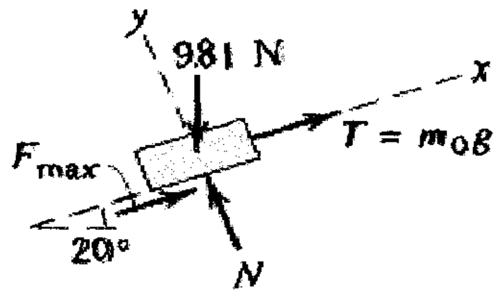
Determine the range of values which the mass mo may have so that the 100 kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surface is 0.30.



Solution:



Case I



Case II

bounded m_o values → block start moving → $F = \mu_s N$

$\left[\sum F_y = 0\right]$ $N - 100g\cos 20 = 0,\ \ N = 922\ N$

Case I: max m_o, start moving up, friction downward

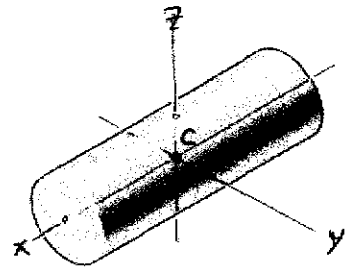
$\left[\sum F_x = 0\right]$ $m_o g - \mu_s N - 100g\sin 20 = 0,\ \ m_o = 62.4\ kg$

Case II: min m_o, start moving down, friction upward

$\left[\sum F_x = 0\right]$ $m_o g + \mu_s N - 100g\sin 20 = 0,\ \ m_o = 6.0\ kg$

$\therefore 6.0 \le m_o \le 62.4\ kg$ and $F \le F_{max} = 277\ N$ up/downward

Center of Mass (CM):



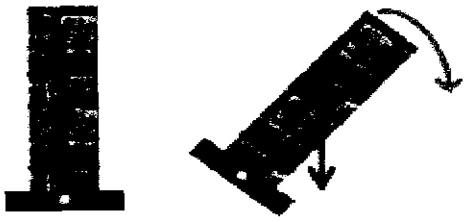
The center of mass is a point which locates the resultant mass of a system of particles or body.

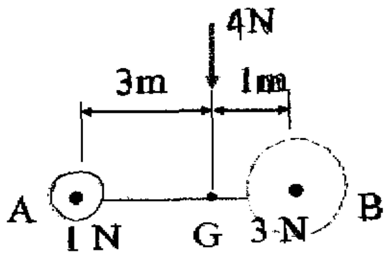
It can be within the object (like a human standing straight) or outside the object like an object of an arc shape.

Center of Gravity:

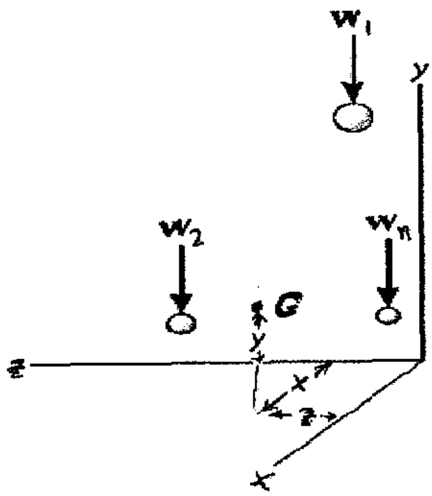
Similarly, the center of gravity (CG) is a point which locates the resultant weight of a system of particles or body. The sum of moments due to the individual particles weights about center of gravity is equal to zero.

when CG extends beyond its support base





Center of Mass and gravity of a System of Particl



Consider a system of n particles as shown in the figure. The net or the resultant weight is given as $W_R = \Sigma W$.

Summing the moments about the y-axis, we get:

$$\bar{x}\, W_R = x_1 W_1 + x_2 W_2 + \ldots + x_n W_n$$

where: x_1 represents x coordinate of W_1, etc..

W: weight of practical

W_R: resultant weight (Total weight)

Similarly, we can sum moments about the x- and z-axes to find the coordinates of G.

By replacing the (W) with a (M) in these equations, the coordinates of the center of mass can be found as the following:

$$\bar{x} = \frac{g\sum m_i x_i}{M \cdot g} \qquad \bar{y} = \frac{\sum m_i y_i}{M}$$

$$\bar{z} = \frac{\sum m_i z_i}{M}$$

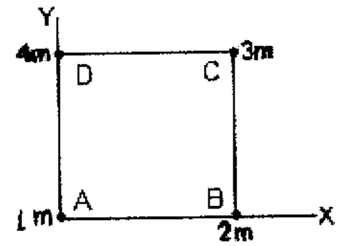
m_i: is the mass of each particle

M: is the sum of the masses of all particles

$\bar{x}_i$, $\bar{y}_i$, $\bar{z}_i$: is the position of each particle with respect to the origin

Example:

corners of a square of side a. Locate the centre of mass.



Solution:

Take the axes as shown in figure. The coordinates of the four particles are as follows.

Particle	Mass	x-coordinate	y-coordinate
A	m	0	0
B	2m	a	0
C	3m	a	a
D	4m	0	a

Hence, the coordinates of the centre of mass of the four particle system are

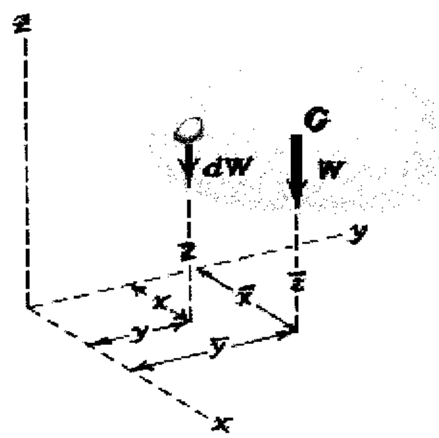
$$X = \frac{m.0 + 2m.a + 3m.a + 4m.0}{m + 2m + 3m + 4m} = \frac{a}{2}$$

$$Y = \frac{m.0 + 2m.0 + 3m.a + 4m.a}{m + 2m + 3m + 4m} = \frac{7a}{10}$$

The centre of mass is at $\left(\frac{a}{2}, \frac{7a}{10}\right)$

Center of Mass and Gravity of an Object

A rigid body can be considered as made up of an infinite number of particles. Hence, using the same principles as in the previous section, we get the coordinates of G by simply replacing the discrete summation sign (Σ) by the continuous summation sign ($\int$) and M by *dm*.



$$\bar{x} = \frac{\int x dW}{\int dW}; \quad \bar{y} = \frac{\int y dW}{\int dW}; \quad \bar{z} = \frac{\int z dW}{\int dW}$$

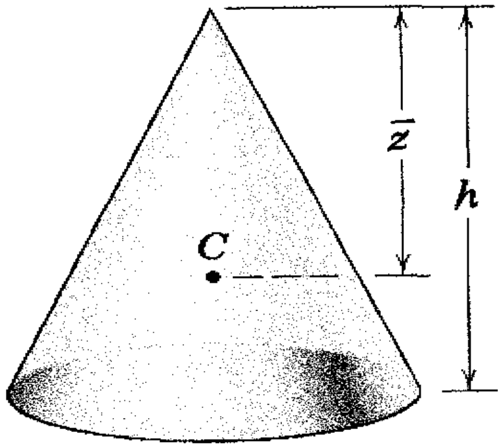
With the substitution of ***W* = *mg*** and ***dW* = *g dm*,** the expressions for the coordinates of the center of gravity become:

$$\bar{x} = \frac{\int x\,dm}{m} \quad \bar{y} = \frac{\int y\,dm}{m} \quad \bar{z} = \frac{\int z\,dm}{m}$$

The density ***ρ*** of a body is its mass per unit volume. Thus, the mass of a differential element of volume ***dV*** becomes ***dm*** = ***ρ dV***. We may then write these expressions as;

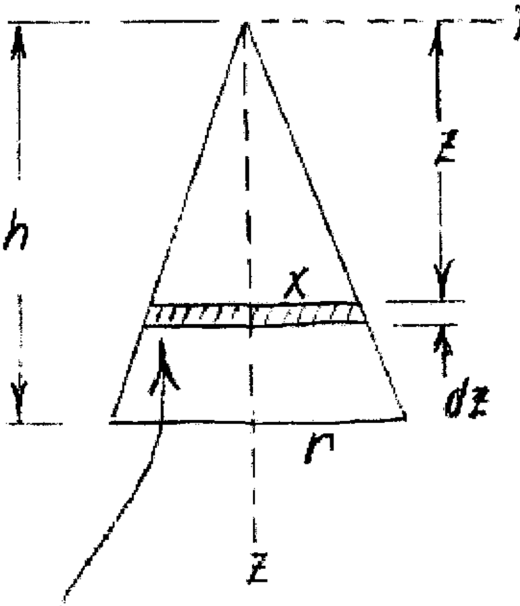
$$\bar{x} = \frac{\int x\rho\,dV}{\int \rho\,dV} \quad \bar{y} = \frac{\int y\rho\,dV}{\int \rho\,dV} \quad \bar{z} = \frac{\int z\rho\,dV}{\int \rho\,dV}, \text{ or } \bar{x} = \frac{\int x\,dV}{V} \quad \bar{y} = \frac{\int y\,dV}{V} \quad \bar{z} = \frac{\int z\,dV}{V}$$

Example: Find the distance from the vertex of the right **circular cone** to the centroid of its volume.



Solution:

$$x = \frac{r}{h} z \,, \quad dV = \pi x^2 \, dz = \pi \frac{r^2}{h^2} z^2 \, dz$$



$$V = \pi \frac{r^2}{h^2} \int_0^h z^2 \, dz = \frac{\pi r^2 h}{3}$$

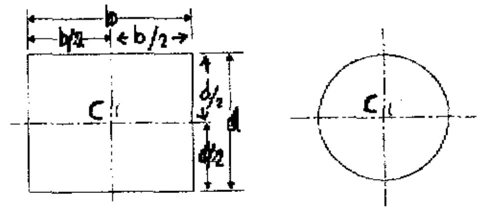
$$\int z \, dV = \pi \frac{r^2}{h^2} \int_0^h z^3 \, dz = \frac{\pi r^2 h^2}{4}$$

$$\bar{z} = \int z \, dV \Big/ V = \underline{3h/4}$$

(Disk - shaped element viewed edge-on.)

Centroid:

The centroid C is a point which defines the *geometric center* of an object. The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogenous (density or specific weight is constant throughout the body). If an object has an axis of symmetry, then the centroid of object lies on that axis.

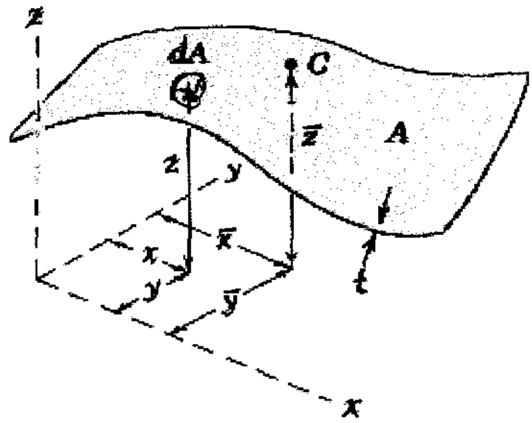


Its location with respect to the origin ($\bar{x}$, $\bar{y}$ and $\bar{z}$) can be determined using the same principles employed to determine the center of gravity of a body. In the case where the material composing a body is uniform or *homogeneous*, the density or specific weight will be constant throughout the body, i.e. When a body of density ρ has a small but constant thickness t, we can model it as a surface area A. The mass of an element becomes $dm = \rho\ t\ dA$. These values will be factored out from the integrals in finding the center of mass and center of gravity and simplifying the expressions, and in this specific case, the centers of mass, gravity and geometry coincide.

$$\bar{x} = \frac{\int x\,dA}{A} \qquad \bar{y} = \frac{\int y\,dA}{A} \qquad \bar{z} = \frac{\int z\,dA}{A}$$

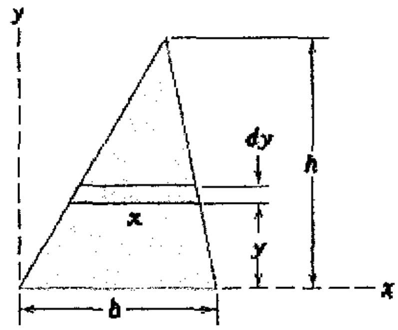
Or;

$$\bar{x} = \frac{\sum A_i x_i}{A}, \quad \bar{y} = \frac{\sum A_i y_i}{A}, \quad \bar{z} = \frac{\sum A_i z_i}{A}$$



Example:

Determine the distance from the base of a triangle of altitude h to the centroid of its area.



Solution: The x-axis is taken to coincide with the base. A differential strip of area $dA = x\,dy$ is chosen. By similar triangles $x/(h - y) = b/h$.

$$[A\bar{y} = \int y_c\,dA]$$

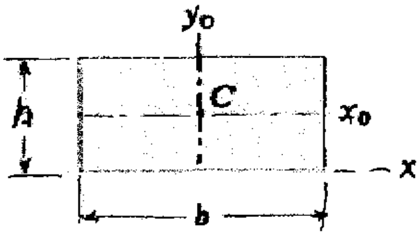
$$\frac{bh}{2}\bar{y} = \int_0^h y\frac{b(h-y)}{h}dy = \frac{bh^2}{6}$$

$$\bar{y} = \frac{h}{3}$$

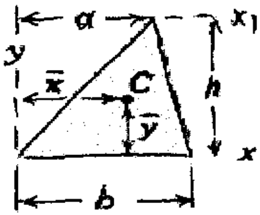
Centroid of some common areas:

FIGURE	CENTROID

Rectangular Area



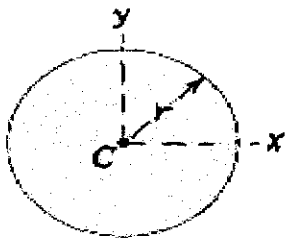
Triangular Area



$$\bar{x} = \frac{a+b}{3}$$

$$\bar{y} = \frac{h}{3}$$

Circular Area



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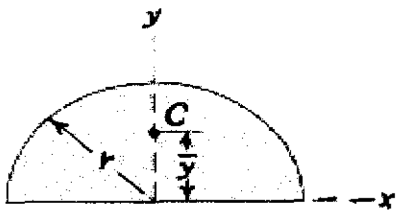
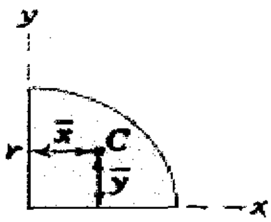
Semicircular Area



$$\bar{y} = \frac{4r}{3\pi}$$

Quarter-Circular Area

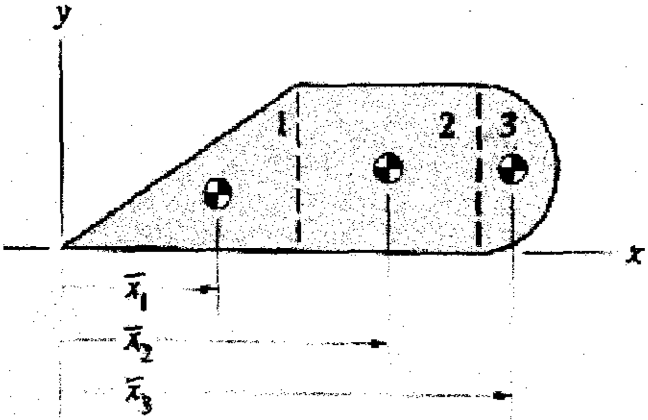


$$\bar{x} = \bar{y} = \frac{4r}{3\pi}$$

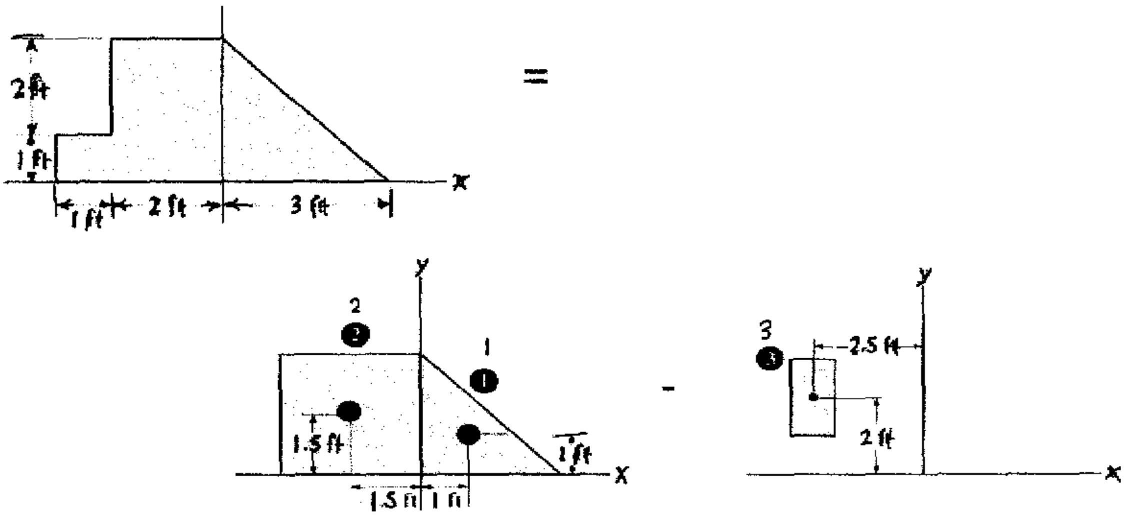
Centroid of composite area:

We can break this figure up into a series of shapes and find the location of the local centroid of each as shown in the following;

$$\bar{x} = \frac{\int_{A_1} x\,dA + \int_{A_2} x\,dA + \int_{A_3} x\,dA}{\int_{A_1} dA + \int_{A_2} dA + \int_{A_3} dA}$$



Example:



Example:

Locate the centroid of the T-section shown in the Fig.

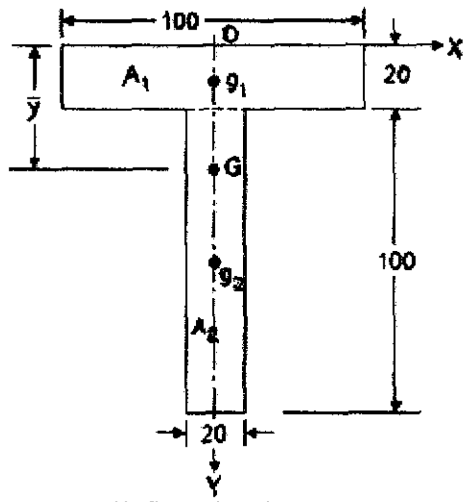
Solution. Selecting the axis as shown in Fig. , we can say due to symmetry centroid lies on **y** axis, *i.e.* $\bar{x} = 0$. Now the given T-section may be divided into two rectangles A_1 and A_2 each of size 100 × 20 and 20 × 100. The centroid of A_1 and A_2 are $g_1(0, 10)$ and $g_2(0, 70)$ respectively.

∴ The distance of centroid from top is given by:

$$\bar{y} = \frac{100 \times 20 \times 10 + 20 \times 100 \times 70}{100 \times 20 + 20 \times 100}$$

$= 40$ mm

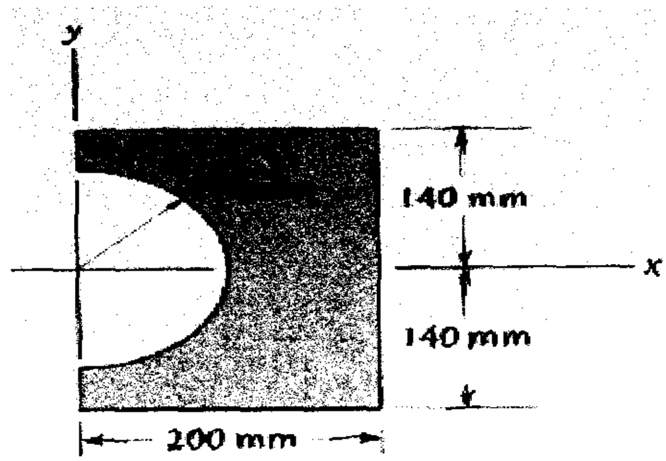
Hence, *centroid of T-section is on the symmetric axis at a distance 40 mm from the top.* **Ans.**



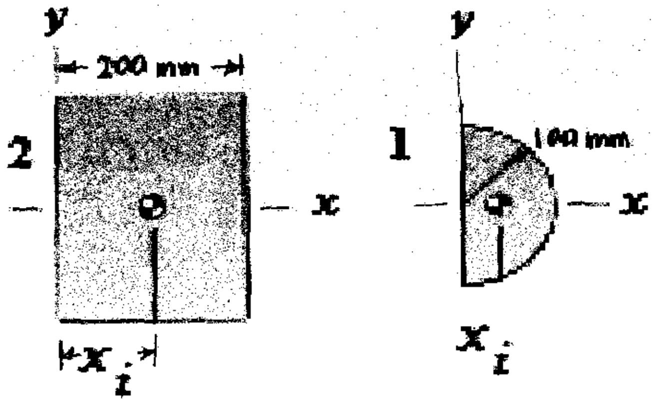
All dimensions in mm

Example:

Determine the centroid of the area



Solution:

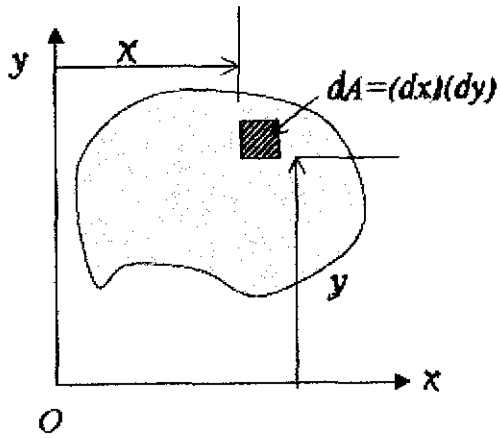


	$\bar{x}_i$	A_i	$\bar{x}_i A_i$
Rectangle	100	(200)(280)	(100)[(200)(280)]
Semi-circle	$\frac{4(100)}{3\pi}$	$-\frac{1}{2}\pi(100)^2$	$-\frac{4(100)}{3\pi}\left[\frac{1}{2}\pi(100)^2\right]$

$$\bar{x} = \frac{x_1 A_1 + x_2 A_2}{A_1 + A_2} = \frac{(100)[(200)(280)] - \frac{4(100)}{3\pi}\left[\frac{1}{2}\pi(100)^2\right]}{(200)(280) - \frac{1}{2}\pi(100)^2} = 122mm$$

Moment of Inertia:

The *moment of inertia* can be considered as a shape factor which indicated how the material is distributed about the center of gravity of the cross-section. It's also defined as the capacity of a cross-section to resist bending. It is usually quantified in m4 or kgm2



So, the moment of inertia has a significant effect on the structural behavior of construction elements. The formulas used for determining the moment of inertia are:

$$I_x = \Sigma y^2 dA \qquad I_y = \Sigma x^2 dA$$

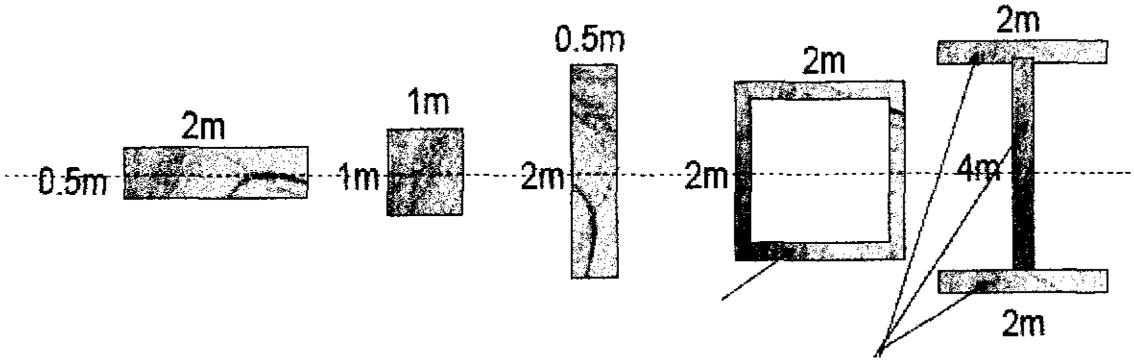
or by using calculus

$$I_x = \int y^2 dA \qquad I_y = \int x^2 dA$$

I_{x2} Moment of inertia about x axis

I_{y2} Moment of inertia about y axis

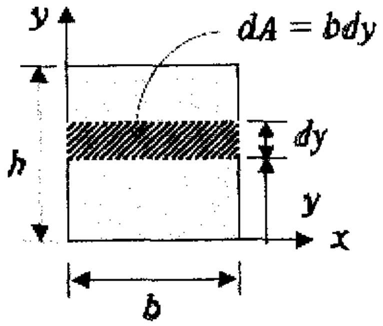
The physical meaning of moment of inertia:



Area	$1m^2$	$1m^2$	$1m^2$	$1m^2$	$1m^2$
Moment of Inertia	$0.02\ m^4$	$0.08\ m^4$	$0.33m^4$	$0.55m^4$	$2.79m^4$

Example:

• Moment of Inertia of a Rectangular Area.

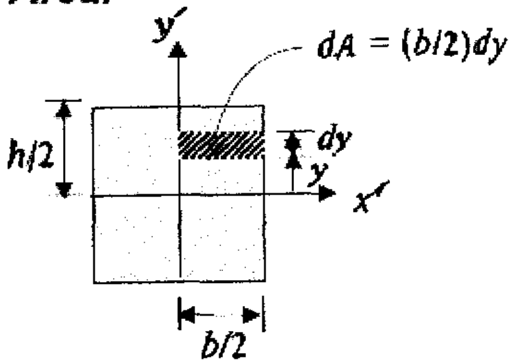


$$I_x = \int_A y^2 dA$$

$$= \int_0^h y^2 (b dy)$$

$$= \left.\frac{(by^3)}{3}\right|_0^h$$

$$= \frac{bh^3}{3} \Leftarrow$$

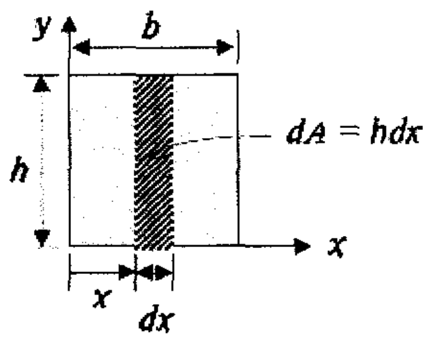


$$\bar{I}_x = I_{x'} = \int_A y^2 dA$$

$$= 4\int_0^{h/2} y^2 (\frac{b}{2} dy)$$

$$= 4(\frac{b}{2}) \left.\frac{y^3}{3}\right|_0^{h/2}$$

$$= \frac{bh^3}{12} \Leftarrow$$

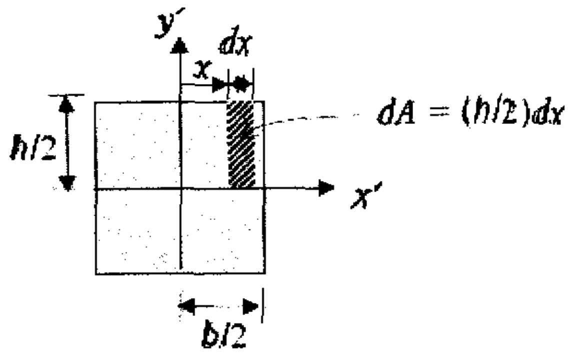


$$I_y = \int_A x^2 dA$$

$$= \int_0^b x^2 (h dx)$$

$$= \left.\frac{(hx^3)}{3}\right|_0^b$$

$$= \frac{hb^3}{3} \Leftarrow$$



$$\bar{I}_y = I_{y'} = \int_A x^2 dA$$

$$= 4\int_0^{h} x^2 (\frac{h}{2} dx)$$

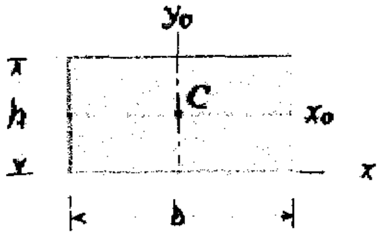
$$= 4(\frac{h}{2}) \left.\frac{x^3}{3}\right|_0^{b/2}$$

$$= \frac{hb^3}{12} \Leftarrow$$

Moment of Inertia of some common areas:

FIGURE	AREA MOMENTS OF INERTIA

Rectangular Area

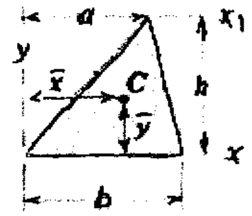


$$I_x = \frac{bh^3}{3}$$

$$\bar{I}_x = \frac{bh^3}{12}$$

$$\bar{I}_z = \frac{bh}{12}(b^2 + h^2)$$

Triangular Area

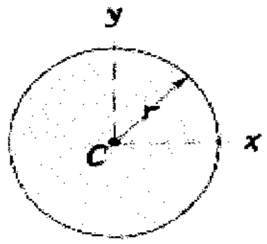


$$I_x = \frac{bh^3}{12}$$

$$\bar{I}_x = \frac{bh^3}{36}$$

$$I_{x_1} = \frac{bh^3}{4}$$

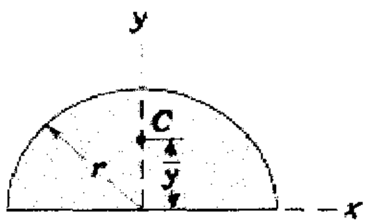
Circular Area



$$I_x = I_y = \frac{\pi r^4}{4}$$

$$I_z = \frac{\pi r^4}{2}$$

Semicircular Area

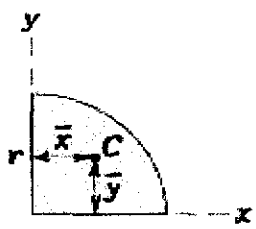


$$I_x = I_y = \frac{\pi r^4}{8}$$

$$\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$$

$$I_z = \frac{\pi r^4}{4}$$

Quarter-Circular Area



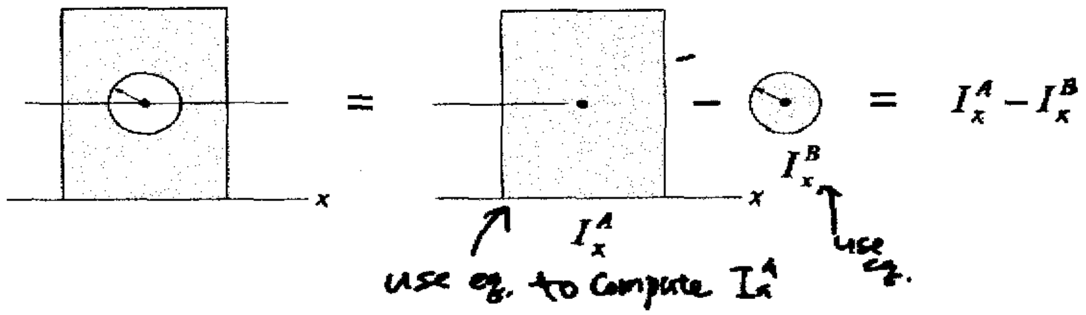
$$I_x = I_y = \frac{\pi r^4}{16}$$

$$\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4$$

$$I_z = \frac{\pi r^4}{8}$$

Moment of inertia of composite areas:

Moment of inertia of a complex area	Break into simpler shapes, then compute moment of inertia of them (using the table)	Add (or subtract) m.o.i. of simple shapes



Parallel Axis Theorem:

If you know the moment of inertia about a **centroidal axis** of a figure, you can calculate the moment of inertia about any **parallel axis to the centroidal axis** using a simple formula;

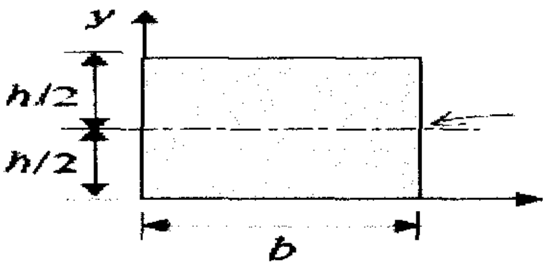
$$I_z = I_{\bar{z}} + Ay^2$$

I_z : moment of inertia around z axis, $I_{\bar{z}}$: moment of inertia around z axis

A : area of the figure

y : distance from the centroid to z axis along y axis

<u>Example</u>: find the moment of inertia using parallel axis theorem



$$\bar{I}_x = \frac{bh^3}{12}$$

$$I_x = \frac{bh^3}{3}$$

$$I_x = \bar{I}_x + Ad^2$$

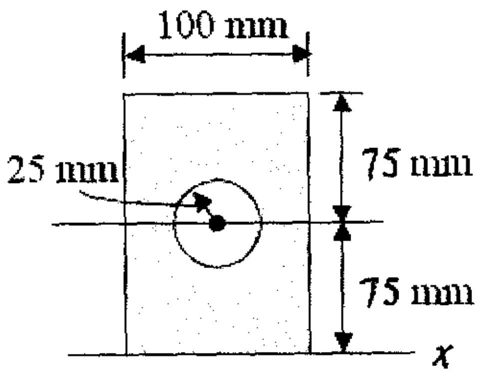
$$= \frac{bh^3}{12} + (bh)\left(\frac{h}{2}\right)^2$$

$$= \frac{bh^3}{12} + \frac{bh^3}{4}$$

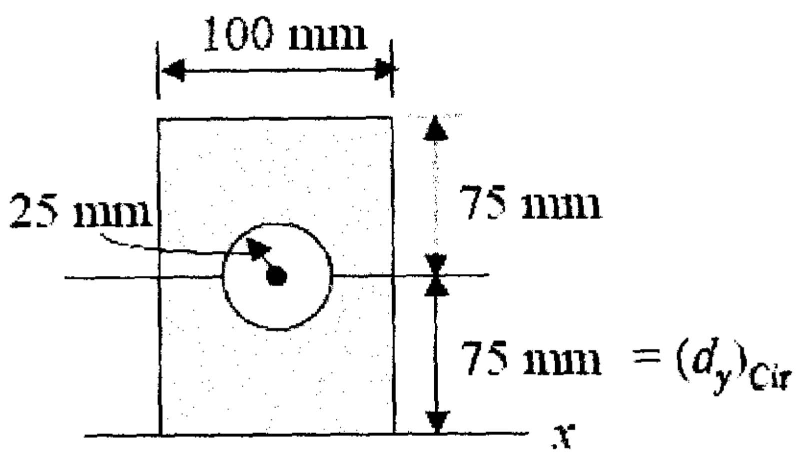
$$I_x = \frac{bh^3}{3}$$

Example:

Compute the moment of inertia of the composite area shown.



Solution:



$$I_x = \left(\frac{bh^3}{3}\right)_{Rect} - (\bar{I}_x + Ad_y^{\,2})_{Cir}$$

$$= \left[\frac{1}{3}(100)(150)^3\right]_{Rect} - \left[\frac{1}{4}\pi(25)^4 + (\pi \times 25^2)(75)^2\right]_{Cir}$$

$$= 101\text{x}10^6 \text{ mm}^4 \Leftarrow$$

Strength of Material

Strength of a material is the ability of that material to withstand an applied stress without failure. Different types of stress can be defined within this field like tensile stress, compressive stresses beside shear stresses. In addition materials could be failed by another types of failures like fatigue stress, thermal stress and/or creep failure. Hence, strength of materials is a subject which deals with loads, deformations and the forces acting on the material.

The main objective of the study of the mechanics of materials is to provide the engineer with the means of analyzing and designing various machines and load bearing structures.

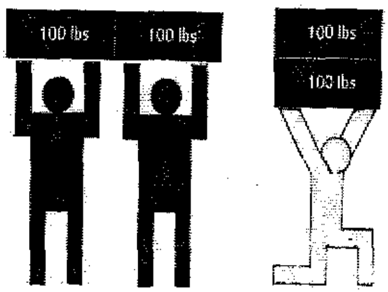
Definitions:

The main objective of the study of the mechanics of materials is to provide the engineer with the means of analyzing and designing various machines and load bearing structures.

Stress: is the internal resistance by a unit area of the material from which a member is made to an externally applied load.

When a force is applied to an elastic body, the body deforms.

The way in which the body deforms depends upon the type of force applied to it.

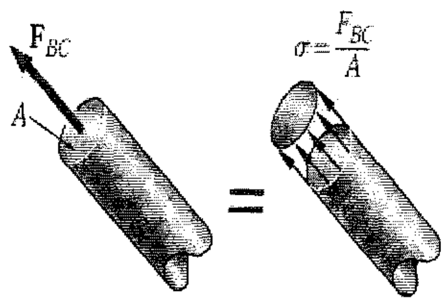


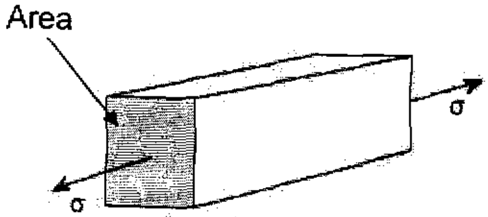
Normal stress:

- normal stress is caused by internal forces that are **perpendicular** to the area considered
- tension in a cable is perpendicular to the cross-sectional area of the cable; this is **normal tensile stress**
- compression in a truss member is perpendicular to the cross-sectional area of the truss member; this is **normal compressive stress**

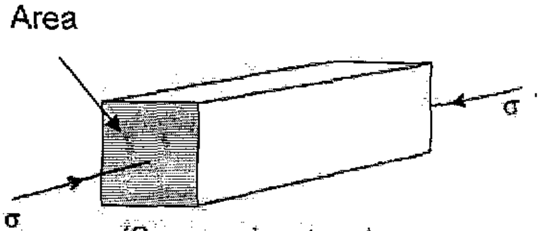
A tensile force makes the body longer.

A compression force makes the body shorter.

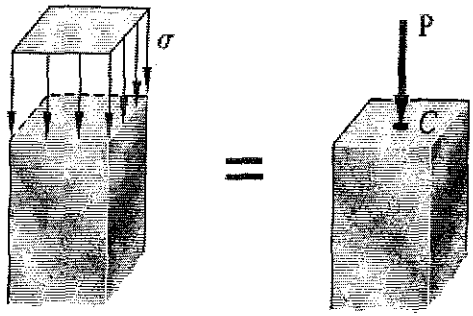


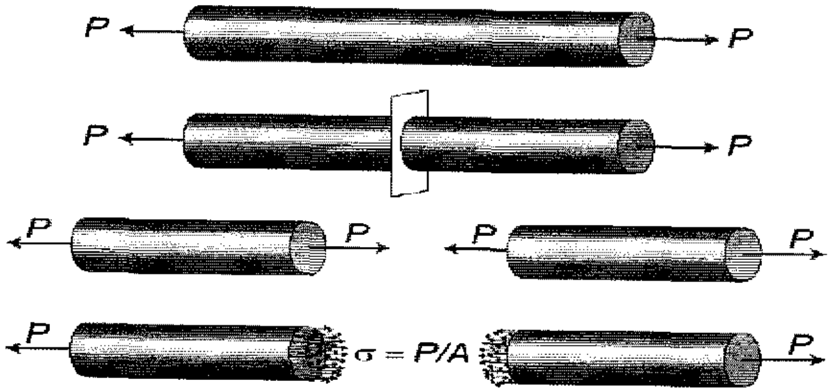


(Tensile stress)



(Compressive stress)





Normal stress is determined using the following equation:

σ = Applied Load/Original Cross Section Area

σ = P/A This has a unit of (Pa) or (N/m^2).

Where σ: stress (Called Sigma)

P: Applied load (Newton or pound force)

A: Area

The units of stress are the units of load divided by the units of area. In the SI system the unit of stress is "Pa" and in the U.S. system it is "Psi". Pa and Psi are related to the basic units through the following relations.

$$1 \quad Pa = \frac{1 \quad N}{1 \quad m^2} \qquad\qquad 10^6\,Pa = 1\,MPa$$

$$1 \quad Psi = \frac{1 \quad lb}{1 \quad in^2}$$

<u>Examples:</u>

Assuming a Force of 500 lbs acts on an area of 10 in^2 the stress will be equal to 500/10= 50 lbs / in^2.

Increasing the area to 20 inches will decrease the stress to 500/20 = 25 lbs / in^2.

If area is doubled the stress will be halved.

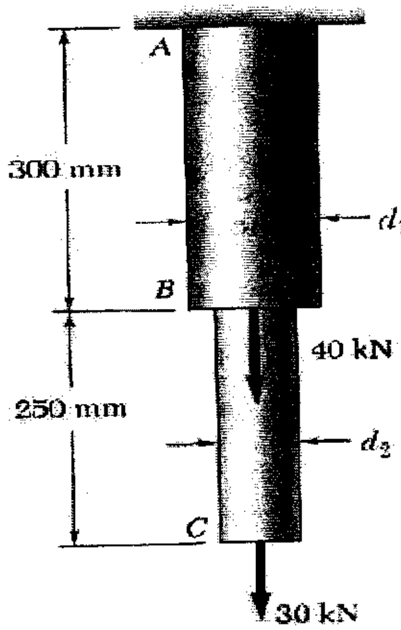
<u>Conversion of MPa and N/mm²:</u>

$$\begin{aligned} 1\ \text{MPa} &= 10^6\ \text{Pa} \\ &= \frac{10^6\ \text{N}}{\text{m}^2} \\ &= \frac{10^6\ \text{N}}{(10^3\ \text{mm})^2} \\ &= \frac{10^6\ \text{N}}{10^6\ \text{mm}^2} \end{aligned}$$

$$1\ \text{MPa} = 1\ \text{N/mm}^2$$

Example:

Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that $d_1 = 50$ mm and $d_2 = 30$ mm, find average normal stress at the midsection of (a) rod AB, (b) rod BC.



a) rod AB:

$$\sigma = \frac{P\ (N)}{A\ (m^2)} \quad Pa$$

$$= \frac{70 * 10^3}{\pi\ (25 * 10^{-3})^2}$$

$$= 35.65 * 10^6 \quad Pa$$

$$= 35.65 \quad MPa$$

b) rod BC:

$$\sigma = \frac{P\ (N)}{A\ (m^2)} \quad Pa$$

$$= \frac{30 * 10^3}{\pi\ (15 * 10^{-3})^2}$$

$$= 42.4 * 10^6 \quad Pa$$

$$= 42.4 \quad MPa$$

Shear stress:

When the material is subjected to a set of equal opposite forces, there is a tendency for one layer of the material to slide over another to produce the form of failure, if this failure is restricted then shear stress (τ) is set up.

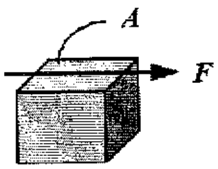
Shear stress is the stress tangent to a surface. If in the following figure the shear stress τ (tau) that results in the shear load (F) is uniformly distributed over the surface, then the shear stress can be calculated by dividing the shear force by the area it is applied on.

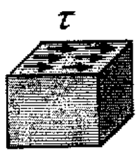
$$\tau = \frac{F}{A}$$

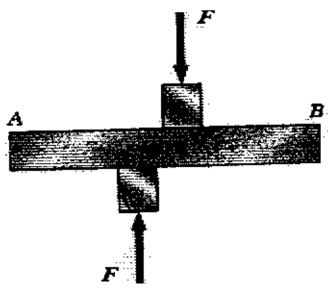
Where T: Shear stress has units similar to normal stress, ***Pa*** or ***N/m2***.

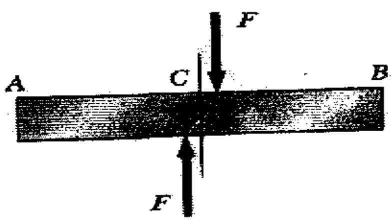
F: Force

A: Shear Area





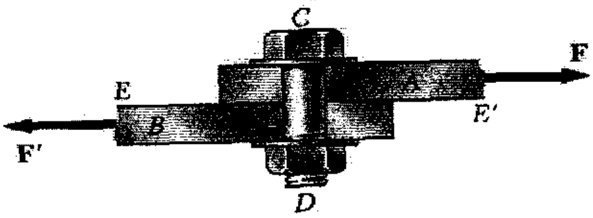


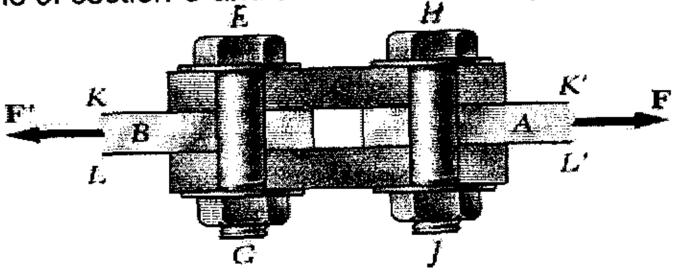


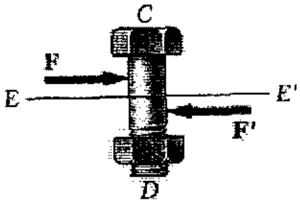
Shearing stresses are commonly found in:

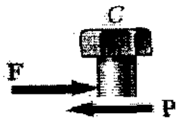
1- Bolts, pins, and rivets.

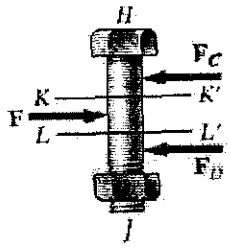
Corresponding internal forces act in the plane of section C and are called *shearing* forces.

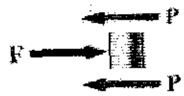


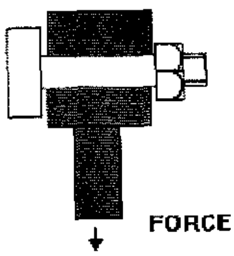


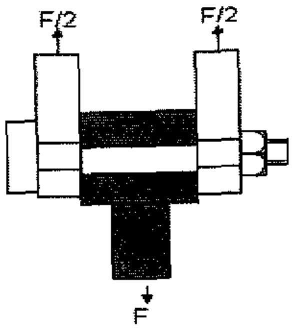


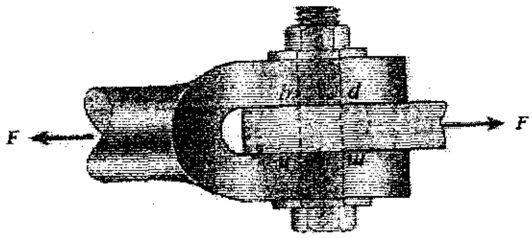










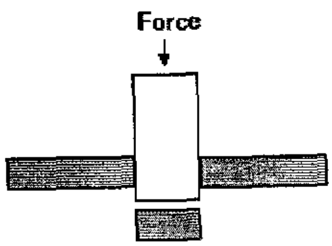


when a pin carries a load.

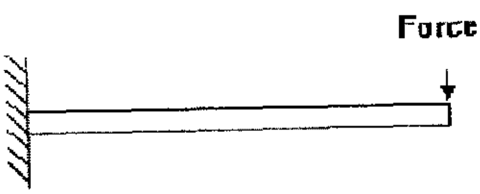
Single Shear $\tau = \dfrac{F}{A}$

Double Shear $\tau = \dfrac{F/2}{A} = \dfrac{F}{2A}$

2- a material is punched:



3- when a beam carries load



Example:

A pin is used to attach a clevis to a rope. The force in the rope will be a maximum of 60 kN. The maximum shear stress allowed in the pin is 40 MPa. Calculate the diameter of a suitable pin.

Solution:

The pin is in double shear so the shear sress is $\tau = \dfrac{F}{2A}$

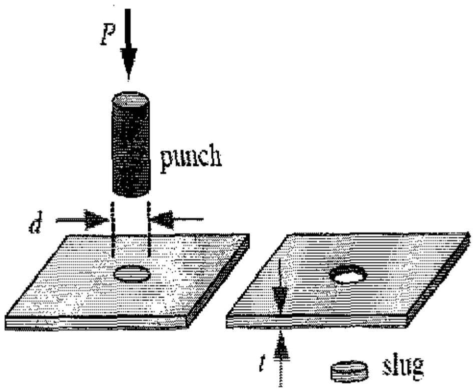
$$A = \frac{F}{2\tau} = \frac{60000}{2 \times 40 \times 10^{6}} = 750 \times 10^{-6}\,\text{m}^2$$

$$A = 750\ \text{mm}^2 = \frac{\pi d^2}{4}$$

$$d = \sqrt{\frac{4 \times 750}{\pi}} = 30.9\ \text{mm}$$

Example:

A 3 mm thick aluminum sheet is cut with a 4 cm diameter round punch. If the punch exerts a force of 6 kN, what is the shear stress in the sheet?



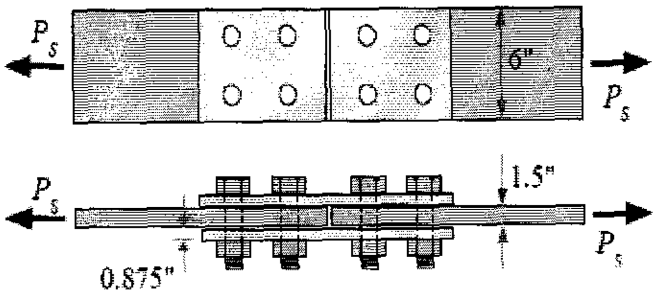
Solution:

The punch will create a round slug, where the cut edge is around the circumference of the slug. Think of the cut edge as the wall of a cylinder with a height of 3 mm and a diameter of 4 cm. The area equals the circumference of the circle times the thickness of the sheet metal:

$$A = \pi\, dt$$

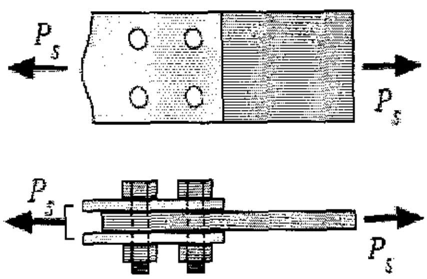
$$\tau = \frac{P}{A} = \frac{P}{\pi\, dt} = \frac{6\,\text{kN}}{\pi \cdot 4\,\text{cm} \cdot 3\,\text{mm}} \left| \frac{100\,\text{cm}}{\text{m}} \right| \frac{10^3\,\text{mm}}{\text{m}} = 15.9\ \text{MPa}$$

Example:



Two A-36 steel plates are joined with two splice plates and eight 7/8 in. diameter A325 bolts. The plates are 6 in. wide and 1.5 in. thick; the splice plates are the same width, and 7/8 in. thick. Bolt threads are excluded from the shear plane. Calculate the load that the bolts can support in order to resist shear failure. Report the result in kips.

Solution From the Bolt Shear Strength table, A325 bolts with threads excluded from the shear plane have a shear strength of 30 ksi. The problem is symmetrical, so either the four bolts in the left plate will fail in shear first, or the four bolts in the right plate will fail first. Therefore, we can erase half of the diagram, and focus on four bolts, and $N = 4$.



The load is carried by two shear planes per bolt, so $n = 2$.

$$P_S = n A_B \tau_{all} N = 2\,\frac{\text{shear planes}}{\text{bolt}} \cdot \frac{\frac{\pi}{4}(0.875\,\text{in.})^2}{\text{shear plane}} \cdot 30\,\frac{\text{kips}}{\text{in.}^2} \cdot 4\,\text{bolts} = 144\,\text{kips}$$

Stress thin Cylinder:

A pressure vessel is a container that holds a fluid (liquid or gas) under pressure, like, propane tanks, and water supply pipes.

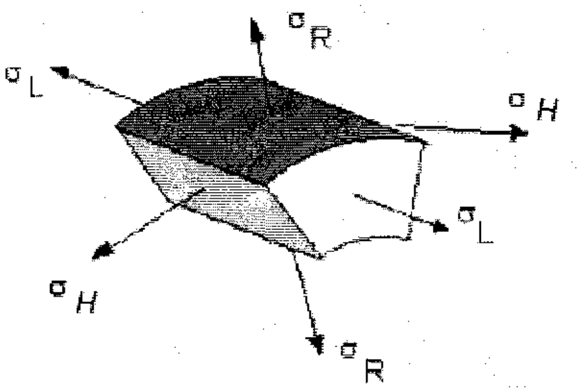
Cylinders can be divided to the thin and thick cylinder. Here we will study the stresses in the thin cylinder (such as pressure vessels). In thin wall cylinders the wall thickness is less than 1/10 of the radius of the container.

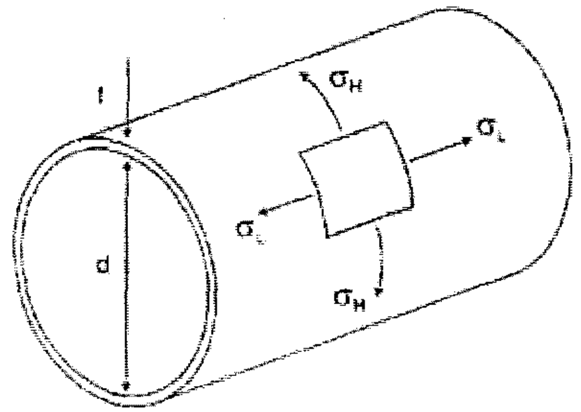
When a thin–walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylinder.

a) Longitudinal stress σ_L.

b) Circumferential or hoop stress σ_H.

c) Radial stress σ_r.

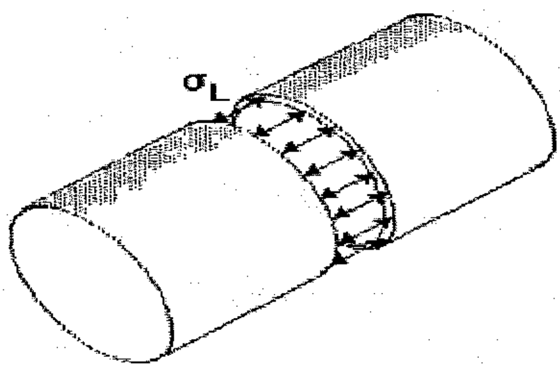




Longitudinal (Axial) stress: is a normal stress parallel to the axis of cylindrical symmetry:

If the pipe has a cap on the end, pressure would push the cap off the end. If the cap is firmly attached to the pipe, then a stress develops along the length of the pipe to resist pressure on the cap.

Imagine cutting the pipe and pressurized fluid transversely. The force exerted by the fluid equals the force along the length of the pipe walls. Pressure acts on a circular area of fluid, so the force exerted by the fluid is:



We can estimate the cross-sectional area of a thin-walled pipe pretty closely by multiplying the wall thickness by the circumference, as shown in fig. a:

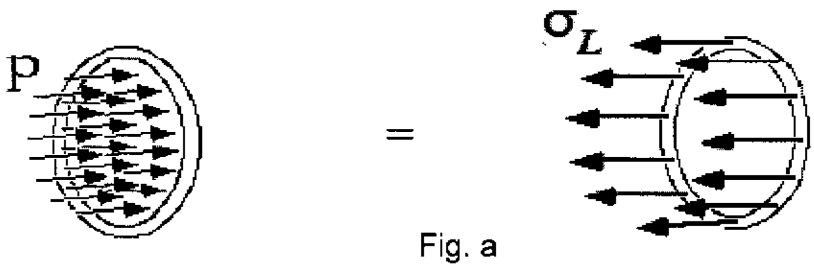


Fig. a

The stress along the length of the pipe is:

Where: σ_L: Longitudinal stress

P: is the internal pressure

R: is the internal radius = d/2 , (where d: is the inside diameter of the cylinder)

t : it the wall thickness

Circumferential stress or hoop stress (σ_H):

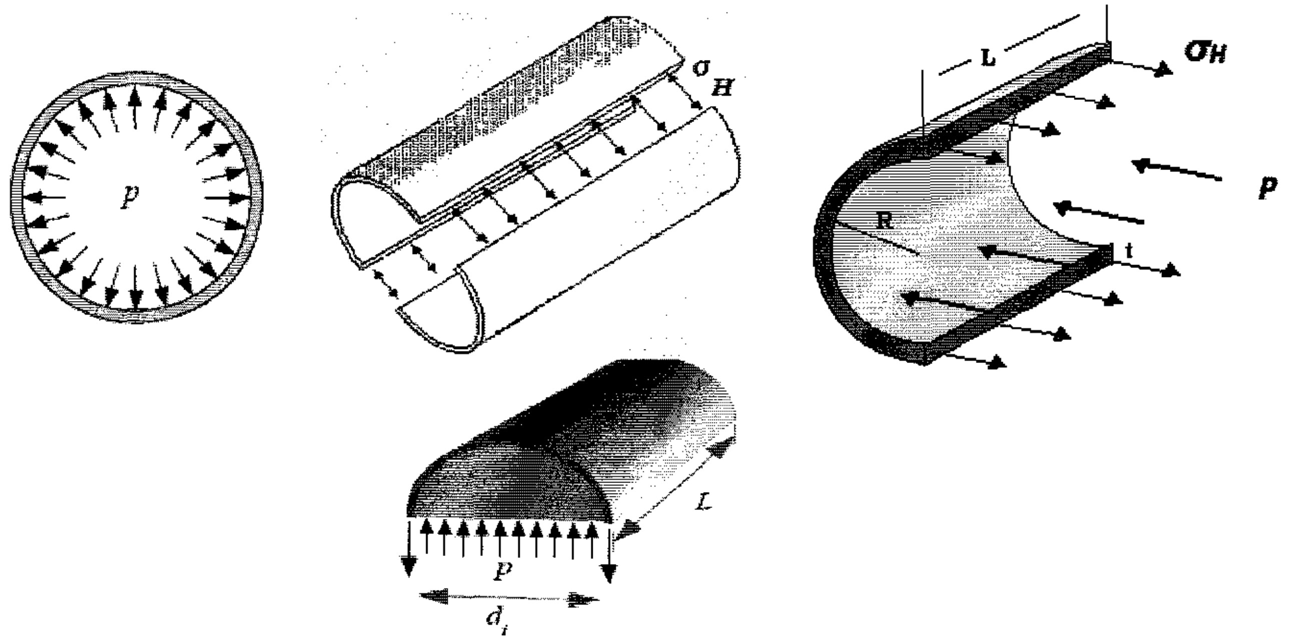


Figure b

Imagine cutting a thin-walled pipe lengthwise through the pressurized fluid and the pipe wall: the force exerted by the fluid must equal the force exerted by the pipe walls (sum of the forces equals zero).

The stress in the walls of the pipe is equal to the fluid force divided by the cross-sectional area of the pipe wall. This cross section of one wall is the thickness of the pipe, t, times its length. Since there are two walls, the total cross-sectional area of the wall is $2t\,L$. The stress is around the circumference or the "hoop" direction,

Notice that the length cancels

where: σ_H : circumferential stress or hoop stress

d_i : is the inside diameter of the pipe, and

L : is the length of the pipe

R : Radius of the pipe

Radial stress: a stress perpendicular to the symmetry axis,

The radial stress for thin vessels is so small in comparison with the hoop and longitudinal stress that it can be neglected. This is obviously an approximation since, in practice; it will vary from zero at the outside surface to a value equal to the internal pressure at the inside surface.

In Inch-pound-second system (IPS) units for P are in pounds-force per square inch (psi). Units for (t), and (d) are in inches (in). SI units for P are in Pascals (Pa), while t and d =2r are in meters (m).

Example: A thin wall pressure vessel is shown in Diagram 3. It's cylindrical section has a radius of 2 feet, and a wall thickness of the 1". The internal pressure is 500 lb/in^2. Determine the longitudinal and hoop stresses in the cylindrical region.

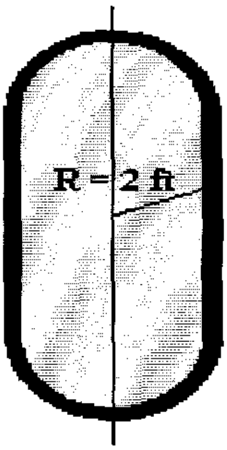


Diagram 3

P = 500 lb/in^2

t = 1"

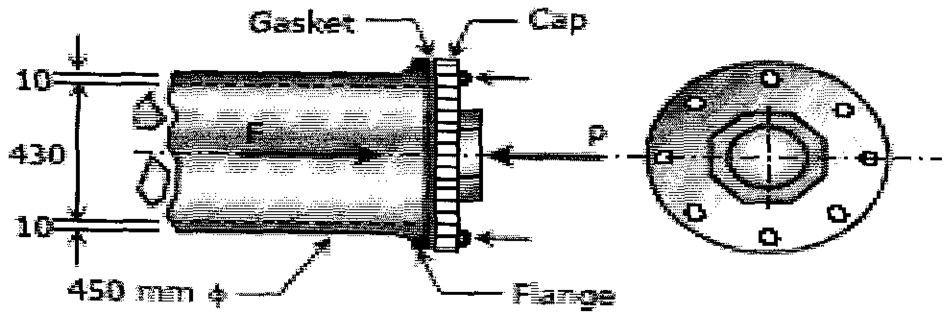
Solution:

We apply the relationships developed for stress in cylindrical:

Example:

A pipe carrying steam at 3.5 MPa has an outside diameter of 450 mm and a wall thickness of 10 mm. A gasket is inserted between the flange at one end of the pipe and a flat plate used to cap the end. How many 40-mm-diameter bolts must be used to hold the cap on if the allowable stress in the bolts is 80 MPa, of which 55 MPa is the initial stress? What circumferential stress is developed in the pipe? Why is it necessary to tighten the bolt initially, and what will happen if the steam pressure should cause the stress in the bolts to be twice the value of the initial stress?

Solution:



$F = \sigma A$

$= 3.5[\frac{1}{4}\pi(430^2)]$

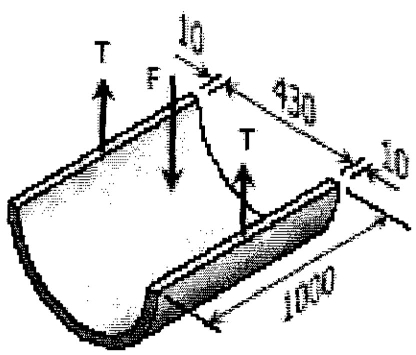
$= 508\ 270.42\ \text{N}$

$P = F$

$(\sigma_{bolt}A)n = 508\ 270.42\ \text{N}$

$(80 - 55)[\frac{1}{4}\pi(40^2)]n = 508\ 270.42$

$n = 16.19$ **say 17 bolts**



Circumferential stress (consider 1-m strip):

$F = pA = 3.5[430(1000)]$

$F = 1\ 505\ 000\ \text{N}$

$2T = F$

$2[\sigma_H(1000)(10)] = 1\ 505\ 000$

$\sigma_H = 75.25\ \text{MPa}$

Discussion:

It is necessary to tighten the bolts initially to press the gasket to the flange, to avoid leakage of steam. If the pressure will cause 110 MPa of stress to each bolt causing it to fail, leakage will occur. If this is sudden, the cap may blow.

Strain

As we load the material, there will be a change in the length of the material.

Strain is the total change in length divided by the original length of the material under tensile or compressive load.

Longitudinal strain is calculated using the following equation:

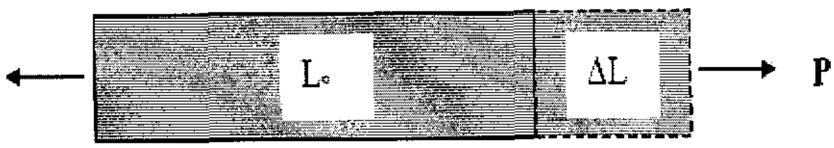
$$\varepsilon = \text{Changing in length/Original length} = \frac{\Delta L}{L_o}$$

Where ε: Strain

L_o: Original length

ΔL: Change in length or (δ)

Unit of Length/Unit of Length] and therefore it is a dimensionless quantity.



Percentage Strain = $\Delta L/L_o$ x 100%

It is to be expected that the tensile stress and strain cause positive increase in length (dimensions), whereas the compressive stress and strain translated to negative change or decrease in dimensions.

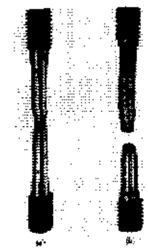
<u>Example:</u>

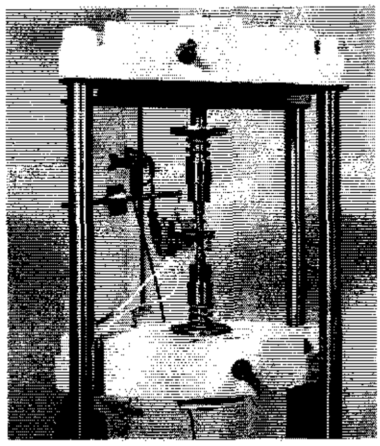
A 6 inch long copper wire is stretched to a total length of 6.05 inches. What is the strain?

Solution:

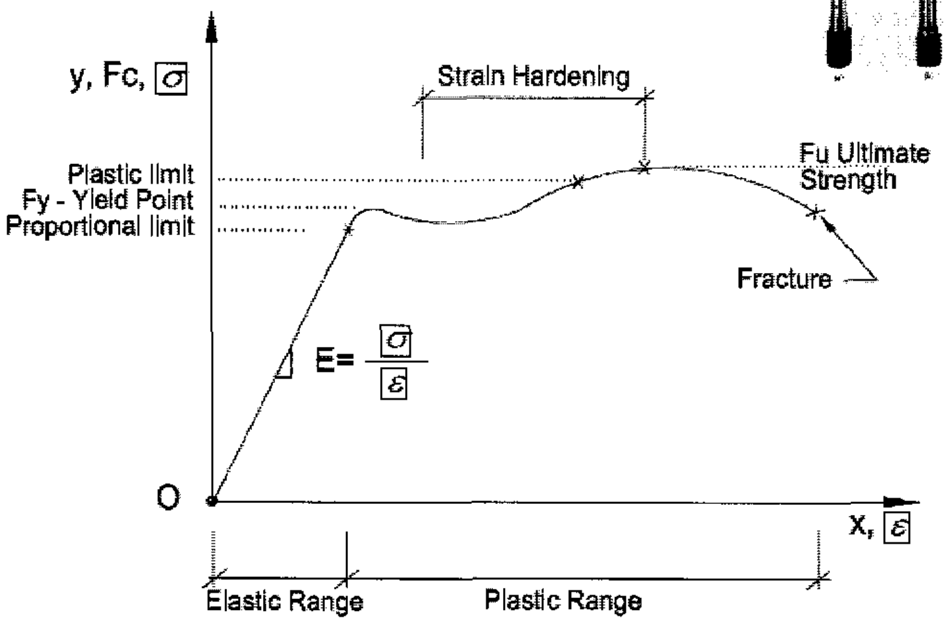
Relationship between stress and strain:

To establish a relationship between stress and strain, we can perform tests on a given specimen, one such test is the tensile test, where the specimen is loaded in tension in a machine (Tensile Test Machine) and the stress and strain are recorded.

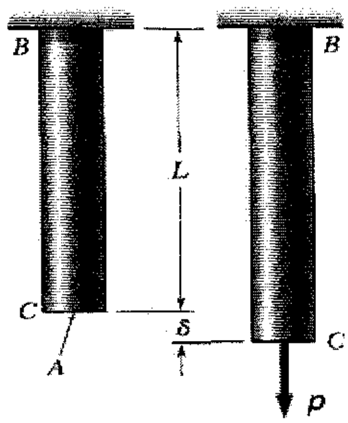




The graph looks like the one in figure bellow:



Stress – Strain curve



The slope of the stress strain curve at the elastic stage is termed the Modulus of Elasticity, or Young's Modulus, in which here is a linear relationship between stress and elongation of a bar in tension.

$\sigma \propto \varepsilon$ Therefore, $\sigma = E \cdot \varepsilon$ $\Longrightarrow$ $E = \dfrac{\sigma}{\varepsilon}$

Where: E Modulus of Elasticity or Young's Modulus.

Substitute the definition of stress

and $E = \dfrac{\sigma}{\varepsilon} = \dfrac{P}{A \cdot \varepsilon}$

Substitute the definition of strain, $\varepsilon = \dfrac{\delta}{L}$ and, $E = \dfrac{P}{A \cdot \varepsilon} = \dfrac{PL}{A\delta} \Longrightarrow \boxed{\delta = \dfrac{PL}{AE}}$

With variations in loading, cross-section or material properties, $\delta = \sum_i \dfrac{P_i L_i}{A_i E_i}$

After the yield the material is in plastic region and if unloaded the material will not return to its original length and will have what is called a permanent deformation or permanent set.

Example:

What tensile stress is required to produce a strain of $8\times10-5$ in aluminum? Report the answer in MPa.

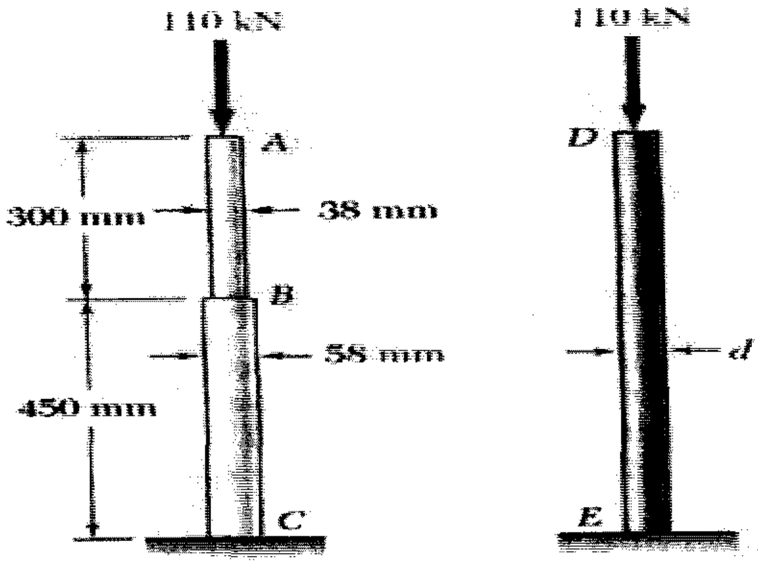
Solution:

Example:

A 70 kN compressive load is lied to a 5 cm diameter, 3 cm tall, steel cylinder. Calculate stress, strain, and deflection.

Example:

The aluminum rod *ABC* ($E = 70$ GPa), which consists of two cylindrical portions *AB* and *BC*, is to be replaced with a cylindrical steel rod *DE* ($E = 200$ GPa) of the same overall length. Determine the minimum required diameter *d* of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 165 MPa.



Solution:

Example:

A metal wire is 2.5 mm diameter and 2 m long. A force of 12 N is applied to it and it stretches 0.3 mm. Assume the material is elastic. Determine the following.

i. The stress in the wire σ.
ii. The strain in the wire ε.

Solution:

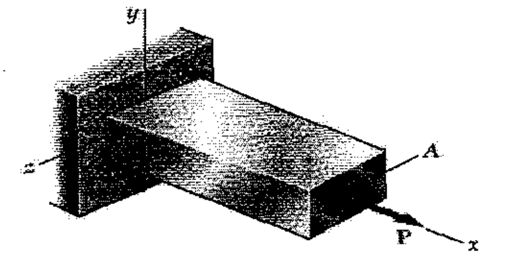
Example:

A Steel column is 3 m long and 0.4 m diameter. It carries a load of 50 MN. Given that the modulus of elasticity is 200 GPa, calculate the compressive stress and strain and determine how much the column is compressed.

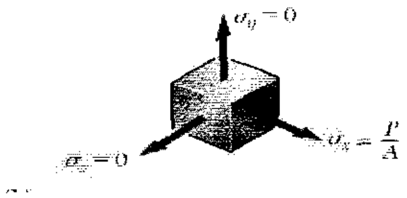
Solution:

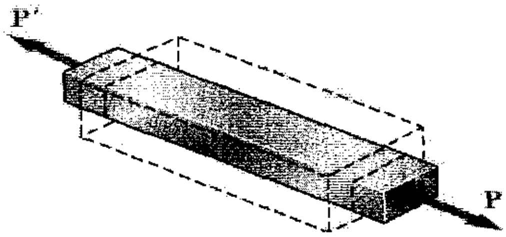
Poisson's Ratio:

$$\nu = -\frac{\text{Lateral strain}}{\text{Longitudinal strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$



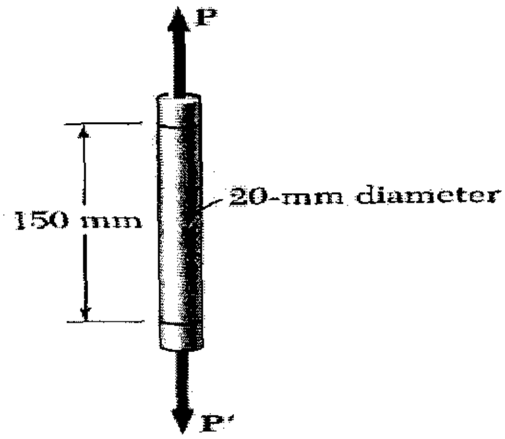
(a)





The axial member shown in the figure, also has a strain in the lateral direction. If the rod is in tension, then stretching takes place in the longitudinal direction while contraction takes place in the lateral direction. The ratio of the magnitude of the lateral strain to the magnitude of the longitudinal strain is called Poisson's ratio ν.

Example: In a standard tensile test, an aluminum rod of 20-mm diameter is subjected to a tension force of $P = 30$ kN. Knowing that $\nu = 0.35$ and $E = 70$ GPa, determine (*a*) the elongation of the rod in an 150-mm gage length, (*b*) the change in diameter of the rod.



Solution:

Example:

A circular aluminum rod 10 mm in diameter is loaded with an axial force of 2 kN. What is the decrease in diameter of the rod? Take $E = 70\ GN/m^2$ and $\nu = 0.33$.

Solution:

Composite Material of Equal length:

Reinforced columns, composite structure of equal length (example pipe inside a pipe) these problems can be solved considering that the change length is same for all materials in that structure.

Example; in reinforced concrete column, steel and concrete length change equally.

Change in length of concrete (δc) = change in length of steel (δs)

$$\delta c = \delta s$$

It is same as equation below for equal length only

$$\varepsilon_C = \varepsilon_s \Longrightarrow \frac{\sigma c}{Ec} = \frac{\sigma s}{Es}$$

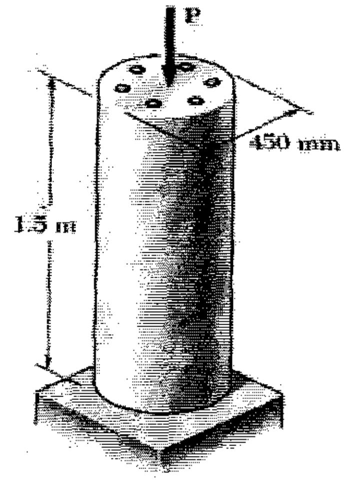
For unequal length it is

$$\frac{\sigma c \,.\, Lc}{Ec} = \frac{\sigma s \,.\, Ls}{Es}$$

Example:

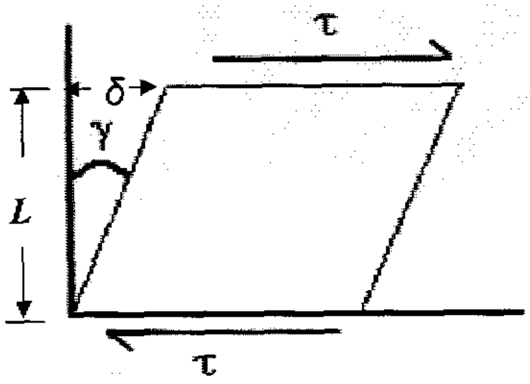
The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550-kN axial centric force **P** is applied to the post.

Solution:



Shear strain:

Consider a rectangular block loaded in shear. The block will distort as a parallelogram, so the top edge moves an amount δ. Divide the distortion by length L perpendicular to the distortion, and you have the shear strain,



$\gamma = \frac{\delta}{L}$ Where γ : Shear strain

δ: Deformation

L: Length

Like normal strain, shear strain is unitless

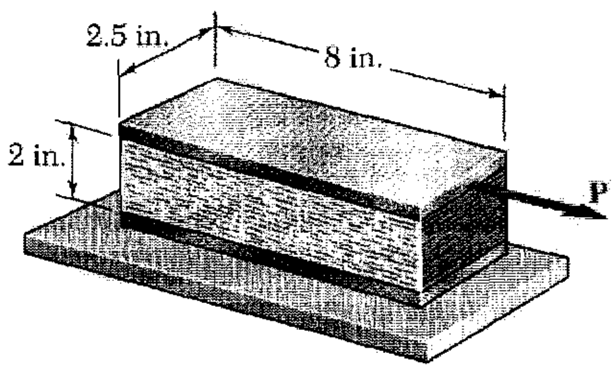
From trigonometry, we know that:

$$\tan \gamma = \frac{\delta}{L}$$

The amount of strain in the figure is exaggerated. For metals, concrete, wood, and most polymers, angle γ is so small that $\tan(\gamma) \approx \gamma$ (in *radians*)

Example:

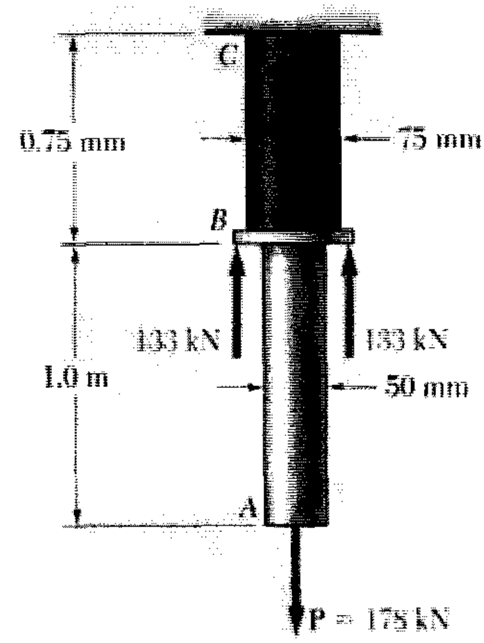
A rectangular block of material with modulus of rigidity G = 90 ksi is bonded to two rigid horizontal plates. The lower plate is fixed, while the upper plate is subjected to a horizontal force P. Knowing that the upper plate moves through 0.04 in. under the action of the force, determine a) the average shearing strain in the material, and b) the force P exerted on the plate.



Example:

Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel ($E = 200$ GPa), and rod BC of brass ($E = 105$ GPa). Determine (*a*) the total deformation of the composite rod ABC, (*b*) the deflection of point B.

Solution:



Key Equations

Normal stress in a tensile or compressive member is the load divided by the cross-sectional area: $\sigma=\frac{P}{A}$

Normal strain is the change in length parallel to the load divided by initial length: $\varepsilon=\frac{\Delta L}{L}=\frac{\delta}{L}$

Young's modulus is the ratio of stress over strain within the elastic zone of the stress-strain diagram: $E=\frac{\sigma}{\varepsilon}$

The change in length of a tensile or compressive member is derived from the three previous equations: $\delta=\frac{PL}{AE}$

Shear stress is the load divided by the area parallel to the load: $\tau=\frac{P}{A}$

Shear strain is the deformation parallel to the load divided by initial length perpendicular to the load: $\gamma=\frac{\delta}{L}$.

Units review

	British	Metric	S.I.
1. Force	Ib, kip, Ton	g, kg,	N, kN
	1 kip = 1000 Ib 1 ton = 2240 Ib	1 kg = 1000 g Ton = 1000 kg	1 kN = 1000 N 1 kg = 10 N
2. Long	in, ft	m, cm, mm	m, cm, mm
	1 f= 12 in	1 m = 100 cm 1 cm = 10 mm 1 m =1000 mm 1 in = 2.54 cm	1 m = 100 cm 1 cm = 10 mm 1 m =1000 mm 1 in = 2.54 cm
3. Stress	psi, ksi $\frac{p}{in^2}, \frac{kip}{in^2}$	Pa ($\frac{N}{mm^2}$), MPa, GPa	

$$\text{MPa} = 10^6\ \text{Pa} = 10^6\ \text{N/mm}^2 \times \frac{1}{1000^2\,\frac{mm^2}{m^2}}$$

$$\text{MPa} = \frac{N}{mm^2}$$

$$\text{GPa} = 10^9\ \text{Pa} = 10^9\ \text{N/mm}^2 \times \frac{1}{1000^2\,\frac{mm^2}{m^2}} = 10^3\,\frac{N}{mm^2} \times \frac{1}{1000\,\frac{N}{kN}}$$

$$\text{GPa} = \text{kN/mm}^2$$